A Multi-Dimensional Correlation Matrix Feature Extraction Technique for Hyperspectral Images

Yang-Lang Chang¹, Hsuan Ren², Jyh-Perng Fang¹, Wen-Yew Liang³, Yun-Ming Liu¹
¹ Department of Electrical Engineering, National Taipei University of Technology
² Center for Space and Remote Sensing Research, National Central University
³ Department of CSIE, National Taipei University of Technology

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Abstract

A novel study of feature extraction technique for hyperspectral images of remote sensing is proposed. The method is based on the greedy modular eigenspace (GME) scheme, which was designed to extract the simplest and the most efficient feature modules for high-dimensional datasets. It presents a framework which consists of two algorithms, referred to as multi-dimensional correlation matrix feature extraction (MD-CMFE) and the feature scale uniformity transformation (FSUT). The MD-CMFE scheme, also known as the complete modular eigenspace (CME), can improve the performance of GME feature extraction optimally by modifying the conventional correlation coefficient operations. It is designed to extract features by a new defined three dimensional correlation matrix (3D-CM) to optimize the modular eigenspace, while FSUT is performed to fuse most correlated features from different spectrums associated with different data sources. The performance of the proposed method is evaluated by applying to hyperspectral images of MODIS/ASTER (MASTER) airborne simulator during the NASA Pacrim II campaign. The experiments demonstrate the proposed MD-CMFE/FSUT approach is an effective tool not only for the feature extraction but also for the feature selection of high-dimensional datasets. It can improve the precision of hyperspectral image classification compared to conventional multispectral classification schemes.

Keywords : greedy modular eigenspace (GME), multi-dimensional correlation matrix feature extraction (MD-CMFE), complete modular eigenspace (CME), feature scale uniformity transformation (FSUT)

1. Introduction

With the evolution of hyperspectral technology, an increasing number of spectral features become available. For the advances of modern remote sensing community, hyperspectral images have become emerging fields with abundances of applications. The demands for higher classification accuracy of remote sensing images have encouraged an increasing usage of the information collected from hyperspectral images. Within the past decade, various hyperspectral image feature extraction and classification techniques have been proposed. Many attentions have been focused on the developing of hyperspectral feature extractions devoted to earth remote sensing. The increment of using high-dimensional data volumes greatly improves the precisions, but provides a challenge for analyzing such high volume datasets. The most widely used techniques are the statistical approaches. One of the well-known methods is orthogonal subspace projection (OSP) [1]. It projects all undesired features into a space orthogonal to the subspace generated by the desired features to extract the most important features and further achieve the high-dimensionality classifications.

In this paper, a new technique is proposed for hyperspectral feature extraction of earth remote sensing. It utilizes the separability of different classes in hyperspectral images to reduce dimensionality and further to generate a unique multi-dimensional correlation matrix feature extraction (MD-CMFE) feature which is developed based on greedy modular eigenspace (GME) scheme [2]. The GME was developed by grouping highly correlated high-dimensional datasets into a smaller subset of modules. MD-CMFE is introduced to improve the performance of feature extraction by modifying the correlation matrix operations. It is designed by a new defined three dimensional correlation matrix (3D-CM) to optimize the modular eigenspace.

The 3D-CMFE is a spectral-based technique that explores the correlation among features. Reordering the feature bands regardless of the original order in terms of wavelength in high-dimensional datasets is an important characteristic of 3D-CMFE. It performs a 3D iteration search algorithm, which reorders the new defined correlation coefficients (CC) in the corresponding new multi-dimensional correlation matrices rows by rows with 3D directions, to group highly correlated features as 3D-CMFE feature eigenspaces. It can be further used for feature extractions. Each ground cover type has a distinct set of 3D-CMFE-generated feature eigenspaces. It selects a subset of non-correlated features using the unique separability of 3D-CMFE embedded between different classes. The proposed 3D-CMFE algorithm provides a fast procedure to select the most significant features for hyperspectral images compared to GME features.

A feature scale uniformity transformation (FSUT) [3] is performed to unify the feature scales of 3D-CMFE of
different classes after finding a 3D-CMFE set. The FSUT selects each single feature by a simple logical AND operation. It can fuse different sources with the most correlated features. It takes advantage of 3D-CMFE to construct correlated features into the most common correlated subspaces. 3D-CMFE/FSUT makes use of 3D-CMFE scheme which tends to equalize all the features in a subgroup with highly correlated variances. It can avoid the bias problems of the traditional principal components analysis (PCA) [4] which transforms the information into linear feature combinations. Our proposed FSUT can mainly reduce computational complexity and further improve classification accuracy as well. It also demonstrates the adaptability and flexibility of the proposed approach for high dimensional datasets. To demonstrate the advantages of the proposed method, we compared several different configurations, which are categorized by using of different feature extraction methods, members of feature and classifiers.

The rest of this paper is organized as follows. In next section, the proposed 3D-CMFE/FSUT method is described in detail. In Section 3, a set of experiments is conducted to demonstrate the feasibility and utility of the proposed approach. Finally, in the last section, several conclusions are presented.

![Figure 1](image)

**Figure 1.** An example illustrating (a) an original GME-CMPM and (b) its corresponding correlation matrix for class \( \omega_k \). (c.) and (d.) The GME set \( \Phi^k \) for class \( \omega_k \) and its corresponding correlation matrix after greedy modular subspaces transformation.

## 2. Methodology

### 2.1 3D-CMFE

In our previous works, we proposed a GME set \( \Phi^k \), which is composed of a group of modular eigenspaces, for the class \( \omega_k \) as shown in Fig. 1. Each modular eigenspace \( \Phi^k \) includes a set of highly correlated features.

A visual correlation matrix pseudo-color map (CMPM) proposed by Lee and Landgrebe [5] to emphasize the importance of second-order statistics in hyperspectral datasets is used to illustrate the magnitude of correlation matrices in our proposed GME scheme [2]. We define a correlation submatrix \( \Phi_{m \times m}^{k} \) which belongs to the \( l \)-th modular eigenspace \( \Phi^l \) of a GME set, \( \Phi^l = (\Phi_1^l, \ldots, \Phi_m^l) \), for a land cover class \( \omega_k \), where \( m \) and \( n_k \) represent respectively the number of feature spaces in modular eigenspaces \( \Phi_i^k \) and the total number of modular eigenspaces of a GME set \( \Phi^k \), i.e. \( l \in \{1, \ldots, n_k\} \). The GME scheme makes use of CCs, the degrees to which two or more quantities are linearly associated, as a reference to constructs the GME. At our convenience, the conventional CC of two dimensions (2D) used by GME is defined as \( c_{2D,i,j} \). Given any pair of variables, \( x_i \) and \( x_j \) for a random vector \( x \), the 2D-CC \( c_{2D,i,j} \) of \( x_i \) and \( x_j \) is defined as

\[
c_{2D,i,j} = \frac{\text{cov}(x_i, x_j)}{\sqrt{\text{var}(x_i) \cdot \text{var}(x_j)}} = \frac{E[(x_i - \bar{u}_i)(x_j - \bar{u}_j)]}{(E[(x_i - \bar{u}_i)^2]E[(x_j - \bar{u}_j)^2])^{1/2}},
\]

where \( \text{cov}(x_i, x_j) \) denotes the covariance of \( x_i \) and \( x_j \), \( \sigma_i \) and \( \sigma_j \) are the variances of \( x_i \) and \( x_j \), \( \bar{u}_i \) and \( \bar{u}_j \) are the means of \( x_i \) and \( x_j \), and \( E \) is the expectation value operation.

The original correlation matrix \( c_{ij}^{E}(m_1 \times m_2) \), where \( m_i \) is the total number of original features,

\[
m_i = \sum_{l=1}^{m_i} m_i, \tag{2}
\]

is decomposed into \( m \) correlation submatrices,

\[
c_{ij}^{E} = c_{11}^{E}(m_1 \times m_1) + \ldots + c_{i}^{E}(m_i \times m_i) + \ldots + c_{nn}^{E}(m_n \times m_n), \tag{3}
\]

to build a GME set \( \Phi^k \) for the class \( \omega_k \). There are \( m! \) possible combinations to construct a candidate GME set. It is computationally expensive to make an exhaustive search to construct a GME set if \( m \) is a large number. In the GME scheme [2], the absolute value of every 2D-CC- \( c_{2D,i,j} \) in the original correlation matrix \( c_{ij}^{E} \) is compared to a threshold value \( tc \) \( (0 \leq tc \leq 1) \). Those adjacent 2D-CC \( c_{2D,i,j} \) that are larger than the threshold value \( tc \) are used to construct a modular eigenspace \( \Phi^l \) in an iterative mode. GME scheme performs a greedy iteration search algorithm which reorders CCs in the data correlation matrix row-by-row and column-by-column to group highly correlated features as GME feature eigenspaces that can be further used for feature extraction as shown in Fig. 1 [2].
The GME was proposed based on the assumption that highly correlated features often appear adjacent to each other in high-dimensional data [6]. This hypothesis may not be true for some modern sophisticated hyperspectral sensor systems. In order to overcome this drawback, we introduce a new 3D-CMFE method to replace the conventional 2D-CC \( c_{ijDc} \) used in GME. The 3D-CMFE extends conventional \( c_{ijD} \) to a new CC \( c_{3D_i,j,k} \), for three dimension (3D) variables of \( x_i \), \( x_j \), and \( x_k \), as defined below

\[
c_{3D_i,j,k} = \frac{\text{cov}(x_i, x_j, x_k)}{(\rho_i^3 \rho_j^3 \rho_k^3)^{1/3}} = \frac{E[(x_i - \mu_i)(x_j - \mu_j)(x_k - \mu_k)]}{(E[(x_i - \mu_i)^3]E[(x_j - \mu_j)^3]E[(x_k - \mu_k)^3])^{1/3}},
\]

(4)

where \( \rho_i^3 \) is a new defined 3D-variance of \( x_i \), the rest of the notations are the same as the conventional correlation coefficients. The number of dimension variables needed to construct a ND-CC \( c_{3D_i,j,k} \) can be determined by the number of hyperspectral features applied to CMFE method for finding an optimal CMFE set \( \Phi^k \). In fact, the conventional 2D-CC \( c_{2D_i,j} \) used in GME is a special case of the new defined 3D-CC \( c_{3D_i,j,k} \) used in 3D-CMFE. The 2D-CMPM used in GME can also be modified to a new 3D correlation matrix pseudo-color cube (CMPC). Compared to the conventional 2D-CMFM in Fig. 1, a three dimensional (3D) visual CMPC is illustrated in Fig. 2. The 3D-CMPC is introduced to represent the magnitude of the new defined 3D-CC \( c_{3D_i,j,k} \) matrices visually for our proposed 3D-CMFE scheme. The 3D-CMFE iteration is initially carried out at 3D-CC \( c_{3D_{i,k}} \in \mathbb{C}^{k \times m} \) for class \( \omega_k \) to construct a 3D-CMPC set \( \Phi^k \). All attributes of \( e_{ij} \) are first set as \( \text{available} \) except 3D-CC \( c_{3D_{i,k}} \). The proposed 3D-CMFE algorithm is as follows:

Step 1. Initialization: A new 3D-CMFE modular eigenspace \( \Phi^k_{\omega} = \Phi^k \) for a class \( \omega_k \) is initialized by a new 3D correlation coefficient \( c_{3D_d,d,d} \), where \( c_{3D_d,d,d} \) and \( d \) are defined as the first available element and its subindex in the 3D-CMPC diagonal list \([c_{3D_{1,1,1}} \cdots c_{3D_{m,m,m-1}}] \) of the original 3D correlation matrix \( e_{ij} \). This 3D diagonal coefficient \( c_{3D_d,d,d} \) is set to \( \text{used} \) and then assigned as the current \( c_{3D_d,d,k} \), i.e., the only one activated at the current time. Then, go to step 2. Note that this 3D-CMFE algorithm is terminated if the last 3D diagonal coefficient \( c_{3D_d,d,d} \) is already set to \( \text{used} \) and the last subgroup \( \Phi^k \) of the 3D-CMFE has been obtained for class \( \omega_k \).

Step 2. If the 3D column subindex \( j \) of the current \( c_{3D_d,j,k} \) has reached the last 3D column \( (i.e., j = m-1) \) in the 3D-CMPC, then a new 3D-CMPC modular eigenspace \( \Phi^k \) is constructed with all \( \text{used} \) 3D correlation coefficients \( c_{3D,d,k} \), these \( \text{used} \) coefficients are then removed from the 3D-CMPC, and the algorithm goes to step 1 for another round to find a new 3D-CMPC modular eigenspace. Otherwise, it goes to step 3.

Step 3. The 3D-CMPC moves the current \( c_{3D_d,j,k} \) to the next adjacent column \( c_{3D_d,j+1,k} \), which will act as the current \( c_{3D_d,j+1,k} \), i.e., \( c_{3D_d,j+1,k} \rightarrow c_{3D_d,j+1,k} \). If the current \( c_{3D_d,j,k} \) is \( \text{available} \) and its value is larger than \( t \), then go to step 4. Otherwise, go to step 2.

Step 4. If \( j \neq d \), swap \( c_{3D_d,j,k} \) with \( c_{3D_d,d,k} \), \( c_{3D_d,d,k} \), and \( c_{3D_d,d,k} \), where the asterisk indicates any 3D row or column subindex in the 3D-CMPC. The attributes of \( c_{3D_d,j,k} \), \( c_{3D_d,d,k} \), and \( c_{3D_d,d,k} \) are then marked \( \text{used} \). Then let \( c_{3D_d,j+1,k+1,d} \), \( c_{3D_d,j+1,k+1,d} \), and \( c_{3D_d,j+1,k+1,d} \in \Phi^k \), where \( j+1 \) means including all 3D coefficients between subindex \( j \) and subindex \( d \). Go to step 2.

A 3D-CMFE \( \Phi^k = (\Phi^k_{\omega_1}, \Phi^k_{\omega_2}, \ldots, \Phi^k_{\omega_n}) \) is eventually composed for the ground cover class \( \omega_k \). For convenience, we sort these 3D-CMFE modular eigenspaces \( \Phi^k_{\omega_1}, \Phi^k_{\omega_2}, \ldots, \Phi^k_{\omega_n} \) according to the number of their feature bands, i.e., the number of feature spaces \( m_1, \ldots, m_t, \ldots, m_n \), in descending order. Each square
(m×m) is filled with an average value of all 3D correlation coefficients inside its 3D correlation submatrix \( \varphi (k) \). Figure 2 illustrates the original 3D-CMFE and the reordered one after a 3D-CMFE transformation. Each land cover class has a distinguishable set of 3D-CMFE feature eigenspaces. Two of these six classes are also shown in Fig. 2.

In this visualization scheme, we can build a 3D-CMFE set efficiently and bypass the redundant procedures of rearranging the band order from the original hyperspectral data sets. Moreover, the 3D-CMFE algorithm can reduce the eigen-decomposition computation significantly compared to the conventional PCA feature extraction. The computational complexity for the conventional PCA is of the order of \( O(m^3) \) and it is \( O(\sum_{i=1}^{m_i} m_i) \) for 3D-CMFE [4]. Furthermore, 3D-CMFE also preserves the original high-correlated information inherited from the sources.

2.2 Feature Scale Uniformity Transformation

After founding a set of 3D-CMFE modules \( \Phi \) for all classes \( \omega_k, k \in \{1, ..., N\} \), a fast, simple and effective FSUT is then performed to fuse the feature scales of these 3D-CMFE sets into an identical 3D-CMFE set \( \Phi_I \) [3]. The FSUT applies logical AND operations to the feature numbers inside each 3D-CMFE modular eigenspace box of different classes to select an identical intersection 3D-CMFE (I3D-CMFE) \( \Phi_I \) set for all classes. A block diagram of 3D-CMFE/FSUT is depicted in Fig. 3.

The FSUT performing a searching iteration to generate an identical I3D-CMFE set \( \Phi_I \) is initially carried out on a newly formed I3D-CMFE feature module \( \Phi_b \), where \( \omega_k \in \{1, ..., n\} \) and \( \Phi_b \in \Phi_i \), in which the first feature \( b_i, \) where \( i \in \{1, ..., n\} \) and \( i = 1 \), of the largest 3D-CMFE module \( \Phi \) is first chosen to form a I3D-CMFE set \( \Phi_I \). Each feature \( b_i \) is assigned an attribute during a FSUT. If the attribute of \( b_i \) is set as available, it means this \( b_i \) has not been yet assigned to any identical 3D-CMFE set \( \Phi_I \). If a \( b_i \) is assigned to a \( \Phi_i \), the attribute of this \( b_i \) is set as used. All attributes of the original \( b_i, i \in \{1, ..., n\}, \) are first set as available. An identical I3D-CMFE set \( \Phi_I \) is composed of a group of identical modular eigenspaces, i.e. \( \Phi_I = (\Phi_{i_1}, ..., \Phi_{i_l}, ..., \Phi_{i_n}) \), for all classes \( \omega_k, k \in \{1, ..., N\} \). Each identical I3D-CMFE feature module \( \Phi_I \) has a unique feature set inside a box in gray. An example of the 3D-CMFE/FSUT in detail is demonstrated in Fig. 4.

For convenience, we sort these I3D-CMFE feature modules \( \Phi_I \) according to the number of their feature values in descending order. In Fig. 4, each I3D-CMFE feature module \( \Phi_i \) has a unique feature set inside a box in gray. The 3D-CMFE/FSUT utilizes the separability of different classes in hyperspectral images to reduce dimensionality and further to generate a unique I3D-CMFE feature. It selects a subset of non-correlated hyperspectral features for hyperspectral images using the unique class separability of I3D-CMFE. Here, we select one of the most correlated features in each I3D-CMFE module \( \Phi_i \), for all class \( \omega_k \), arbitrarily to compose an identical I3D-CMFE set \( \Phi_I \). The 3D-CMFE/FSUT algorithm provides a quick feature extraction technique and an instant feature selecting procedure to find the most significant hyperspectral features compared to the conventional feature extraction methods.

Figure 3. The block diagram of 3D-CMFE/FSUT.

Figure 4. An example of the proposed 3D-CMFE/FSUT approach. The I3D-CMFE modules \( \Phi_i \) are obtained in the final column (inside the gray bold 3D-CMFE modular eigenspace boxes).

Figure 5. The Au-Ku test site.
3. Experiments

A plantation area in Au-Ku on the east coast of Taiwan shown in Fig. 5 was chosen for investigation. The image data were obtained by the MODIS/ASTER (MASTER) airborne simulator instruments as part of the PacRim II project [7]. A ground survey was made of the selected six land cover types at the same time. The proposed 3D-CMFE/FSUT method was applied to 35 features selected from the 50 contiguous features (excluding the low signal-to-noise ratio mid-infrared channels) of MASTER [7]. Six land cover classes were used in the experiments. One hundred and fifty labeled samples were randomly collected from ground survey data sets by iterating every fifth sample interval for each class. Thirty labeled samples were chosen as training samples, while the rest were used as test samples, i.e. the samples were partitioned into 30 (20%) training and 120 (80%) test samples for each test case. Three CC threshold values, tc =0.75, 0.80 and 0.85, were selected to carry out the 3D-CMFE. Three fixed I3D-CMFE feature modular features (three, four and five) were chosen for each class. The criterion for calculating the classification accuracy of experiments was based on exhaustive test cases. Finally, the accuracy was obtained by averaging all of the combined results as described above.

To demonstrate the advantages of 3D-CMFE/FSUT method, three configuration groups were compared in Fig. 6. The first group was for 3D-CMFE. In this case, the 3D-CMFE was applied to four different combinations (three, four, five and six) of six ground cover classes in MASTER to generate the I3D-CMFE. One feature was arbitrarily selected from each I3D-CMFE feature module to decompose the Euclidean distances and apply to minimum distance classifier. For the second group, the same datasets were used as for the first group. The Euclidean upper bounds, average bounds and lower bound represent respectively the best upper bound, the average features (between upper bound and lower bound) and the worst lower bound feature features obtained from original datasets described in [7]. The third group, Euclidean PCA, stands for the primary principal components of PCA from original datasets. Compared to the second and third groups, our proposed 3D-CMFE has the best accuracy.

4. Conclusions

This paper presents a new feature extraction technique to hyperspectral images. The 3D-CMFE/FSUT makes use of the 3D-CMFE developed by grouping highly correlated features into a small set of features regardless of the original order of wavelengths. The FSUT applies intersection operations to the feature numbers inside each 3D-CMFE module to extract the feature scales of 3D-CMFE and construct an identical I3D-CMFE feature module set. With this particular characteristic, 3D-CMFE/FSUT can be implemented as an excellent feature extractor as well as a distinguished feature selector to generate the most significant feature set for each classes presented in the hyperspectral images and high dimensional datasets. The features selected by 3D-CMFE/FSUT contain discriminatory properties which are crucial to subsequent classifications. It makes use of the potential significant separability inherited from 3D-CMFE to find a unique set of the most important features with little computational complexity. It can not only well perform the feature extractions but also speed up the following distance decompositions. Experimental results demonstrate that the proposed 3D-CMFE/FSUT is very effective and can also serve as an alternative for the feature selections.

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6. References


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**摘要**

本文提出一個新穎的特徵抽取技術，應用於在『高光譜』感測影像(hyperspectral images)上。本方法為架構在之前我們所提之『貪婪模組特徵空間』(greedy modular eigenspace - GME)方法之上，GME 方法是以抽取高維資料中最「簡單」且最有「效率」的特徵模組而著名。本方法架構在兩個演算法之上，分別為『三維度的相關係數矩阵特徵抽取』(three dimensional correlation matrix feature extraction 3D-CMFE)法及『特徵尺度齊一化轉換』(feature scale uniformity transformation - FSUT)法。3D-CMFE 方法;又稱為完整模組特徵空間 (complete modular eigenspace - CME)，能夠以一個特殊修改的相關係數運算，有效地改進 GME 抽取特徵最佳化的效能。本方法設計一個新定義的 3D 相關係數矩陣，用以抽取最佳化特徵空間模組。同時在配合 FSUT 方法，將本方法所計算出之不同物種分類的最佳化光譜資料特徵空間模組，來聚合成為一個可共用之較高的相關係數之特徵維度，將其對應的不物種光譜之『相關矩陣』群，組成多個特徵尺度齊一之『貪婪模組特徵空間』，進而改良並增進遙測影像的分類辨識率。利用此演算法技術，可以將光譜解析度特性加以開發應用，並可充分利用其絕佳之分散性，提供一個很好的特徵抽取方法。最後並藉由實際校正過後的美國 Pacrim II 計畫所提供之完整台灣『高光譜』遙測影像資料，以實地測量的台灣地表真實資料為基準，用以驗證本文所提之方法的正確性，並與傳統遙測影像資料分類方法作一比較，印證了本3D-CMFE/FSUT 方法非常適用於『高維資料』的『特徵抽取』及『特徵選擇』(feature selection)的特性。

**關鍵字**：貪婪模組特徵空間(GME), 多維度的相關係數矩陣特徵抽取(MD-CMFE), 完整模組特徵空間(CME), 特徵尺度齊一化轉換(FSUT)。