A NEW SINGLE TONE DETECTION ALGORITHM

Li-Te Shen
Department of Electrical Engineering
National Taipei University of Technology
Taipei, Taiwan, R.O.C.
lidshen.j292105@msa.hinet.net

Shaw-Hwa Hwang
Department of Electrical Engineering
National Taipei University of Technology
Taipei, Taiwan, R.O.C.
hsf@ntut.edu.tw

Abstract—In the paper, the multi-correlation algorithm and the quadrate-carrier structure (MCA-QCS) are proposed to detect the single tone. The detection output with band-active response is achieved. The bandwidth and the carrier frequency of the band-active function can be easily set in the MCA-QCS. The computation requirement is dramatic low and the frequency response is superior to the other algorithm, such as Goertzel algorithm, single-correlation algorithm, IIR filter, and FIR filter. Average two multiplies and two adders are needed for each sample only. The experiment result of DTMF detection for telephone line is analyzed. Those results confirm that the performance on computation and frequency response for the new algorithm is superior to Goertzel algorithm.

Keywords: Goertze algorithm, Tone detection

I. INTRODUCTION

Single tone detection method is popular for many applications. In the traditional personal communication system (PCS), such as the PSTN system, the single tone detection method is widely used on the detection of the DTMF signal, the dial tone, the busy tone, the ring-back tone, the disconnect tone, and the caller-ID tone.

In the past, many algorithms are proposed for the single tone detection. The FFT with spectrum peak detection is a well-known approach for the single or multi-tone detection. In [5], various FFT methods for single tone detection are evaluated and compared in detail. However, the FFT method calculates all components over the frequency domain exhaustively. But only one or two components on the frequency domain need to be calculated in the single tone detection process.

In [1], Goertzel proposed the Goertzel-algorithm for single tone detection. The individual DFT coefficient is generated using a simple recursive filter and a second-order digital resonator. The complexity is reduced dramatically and cost-down on device is achieved. Thus, many solutions [7] for single tone detection have great interest on the Goertzel-algorithm.

In the general communication, the QAM method is used to detect the carrier-frequency and recover the binary message. In this approach, the single-correlation algorithm and quadrate-carrier structure (SCA-QCS) is employed to detect the single tone. The computation requirement is dramatic low. Average 2 multiplies and 2 adders are need only for each sample. Total 0.032MIPS is needed while the sample rate is 8 kHz. Moreover, the quadrate-carrier structure can avoid the variant effect contributed from the phase offset. However, the frequency response of SCA-QCS is a narrow-band discriminator and is similar to the $\text{Sinc}(f)$ function. The bandwidth of this narrow-band discriminator is very small and is dependent on the frame length of correlation method. In a generic single tone detection case, such as DTMF detection in ITU-T standard, the acceptable region is less than 1.5%. The rejected region is great than 3.5%. The region between 1.5% and 3.5% can be accepted or rejected. Thus, the SCA-QCS method can’t be easily adjusted to match any specification of single tone detection.

In this article, a multi-correlation algorithm with quadrate-carrier structure (MCA-QCS) is proposed to detect the single tone. In the MCA-QCS method, the computation requirement is similar to the SCA-QCS method and is dramatic low. However, the frequency response is superior to the SCA-QCS, FIR filter, or the IIR filter. The bandwidth of the narrow-band discriminator, the carrier-frequency, and the frame length of correlation can be easily set in the MCA-QCS method. The MCA-QCS method is suitable for any specification of single tone detection. The detail description on the MCA-QCS method will be given in the following section. Moreover, the experiment results of DTMF detection will be given in the next section. The conclusion will be given in the last section.

II. THE MCA-QCS METHOD

In this paper, a new single tone detection algorithm is proposed. The multi-correlation algorithm with the quadrate-carrier structure (MCA-QCS) is employed to improve the performance of single tone detection and to reduce the computation requirement. The quadrate-carrier structure is employed to remove the variant effect contributed from the offset of phase. The $\cos(f_0)$ and $\sin(f_0)$ are the multi-correlation tables estimated from a sophisticated process. The frequency $f_0$ is the carrier frequency of the band-active function. The detection output $y(f_0)$ is a band-active value defined as below:
\[
y(f, 0) = \left[\left(\sum_{n=0}^{L-1} x[n] \cdot \cos(\frac{2\pi f_0 n}{L})\right)^2 + \sum_{n=0}^{L-1} x[n] \cdot \sin(\frac{2\pi f_0 n}{L})^2\right]^{0.5}
\]

(1)

The output value \(y(f, 0)\) is estimated from each frame. The \(L\) is the frame length of input signal. The \(\cos_{f_0}\) and \(\sin_{f_0}\) are the multi-correlation tables defined as below:

\[
\cos_{f_0}[n] = \sum_{f=f_0-2W}^{f_0+2W} \cos\left(\frac{2\pi f n}{2W}\right) \cdot G(f)
\]

\[
\sin_{f_0}[n] = \sum_{f=f_0-2W}^{f_0+2W} \sin\left(\frac{2\pi f n}{2W}\right) \cdot G(f)
\]

(2)

The \(G(f)\) is the energy factor for the \(\cos_{f_0}\) and \(\sin_{f_0}\) tables. The \(W\) is the bandwidth of the band-active function. If the \(W\) is equal to zero, the Eq.1 is a generic correlation function. The \(G(f)\) is the most important factor who make the Eq.1 be a frequency discriminator with band-active. The detection response will be active within the frequency interval \([f_0 - W, f_0 + W]\). However, the frequency components of the \(\cos_{f_0}\) and \(\sin_{f_0}\) tables are located in \([f_0 - 2W, f_0 + 2W]\). Moreover, in the Eq.1, the average computation requirement of one sample is \(2 - 1/L\) for adder and \(2 + 2/L\) for multiply. It is dramatic low.

In the Eq.2, the multi-correlation table \(\cos_{f_0}\) and \(\sin_{f_0}\) are derived from the sophisticated process. In this process, the single-correlation response for a single tone signal \(\cos(2\pi f_0 t + \theta)\) at carrier frequency \(f_0\) is \(A_{f_0}(f_0)\) and is defined as below:

\[
A_{f_0}(f_0) = \int e^{j2\pi f_0 t} \cdot \cos(2\pi f_0 t + \theta) dt
\]

(3)

In the generic single tone detection, \(A_{f_0}(f_0)\) is expected to be an ideal band-active function at \([f_0 - W, f_0 + W]\). Thus, the MCF can be defined as below:

\[
B_{f_0}(f_0) = \int_{f_0-2W}^{f_0+2W} A_{f_0}(f_0) \cdot G(f_0) df_0
\]

(4)

The \(G(f_0)\) is the gain factor for the single-correlation function \(A_{f_0}(f_0)\). The amplitude of \(B_{f_0}(f_0)\) is expected to be an ideal band-active function \(\Pi\left(\frac{f_0 - f_0^*}{2W}\right)\). Thus, the Eq.4 can be re-written as below:

\[
\Pi\left(\frac{f_0 - f_0^*}{2W}\right) = B_{f_0}(f_0)B_{f_0}^*(f_0)
\]

\[
= \int_{f_0-2W}^{f_0+2W} A_{f_0}(f_0)A_{f_0}^*(f_0)G(f_0)G^*(f_0) df_0 dc_0
\]

(5)

In the real case, a digital system is employed to implement the single tone detection algorithm. Thus, the Eq.5 must be re-written as below:

\[
\sum_{k=0}^{N-1} A_{k,0}[k]G[k] = \sum_{k=0}^{N-1} A_{0,0}^*[k]G^*[k] = \sum_{k=0}^{N-1} \hat{A}_k[k]G[k] = \Pi\left(\frac{k - N/2}{2W}\right)
\]

(6)

The digital function \(A_{k,0}[k]\) is defined as below:
\[ A_{k\theta}[k1] = \sum_{n=0}^{L-1} e^{2\pi j \frac{n k W}{N} + f c - 2W} \]  

(7) \[
\cos\left(\frac{2\pi n k W}{SR} \left(\frac{4kW}{N} + f c - 2W + \theta\right)\right) 
\]

The value \( L \) is the time interval or the frame length of the correlation function in the digital form. It is corresponding to the value \( Du \) in the analog form. The \( W \) is the bandwidth of the band-active function. The \( N \) is the length of the function \( A_{k\theta}[k1] \). It means that the value of \( k \) and \( k1 \) is located into \([0,N-1]\). The \( fc \) is still the carrier-frequency of the band-active function. The \( \hat{A}_{k1,k2} \) can be defined and re-written as below:

\[ \hat{A}_{k1,k2} = RA_{k1,k2} + j\hat{A}_k[k1,k2] \]  

(8) 

Thus, the Eq.6 can be re-written in the matrix form listed as below:

\[
\begin{bmatrix}
\hat{A}_{[0,0]} & \hat{A}_{[0,1]} & \ldots & \hat{A}_{[N-1,N-2]} & \hat{A}_{[N-1,N-1]} \\
\hat{A}_{[1,0]} & \hat{A}_{[1,1]} & \ldots & \hat{A}_{[N-1,N-2]} & \hat{A}_{[N-1,N-1]} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\hat{A}_{[N-2,0]} & \hat{A}_{[N-2,1]} & \ldots & \hat{A}_{[N-1,N-2]} & \hat{A}_{[N-1,N-1]} \\
\hat{A}_{[N-1,0]} & \hat{A}_{[N-1,1]} & \ldots & \hat{A}_{[N-1,N-2]} & \hat{A}_{[N-1,N-1]} \\
\end{bmatrix}
\begin{bmatrix}
\hat{G}_{[0,0]} \\
\hat{G}_{[0,1]} \\
\vdots \\
\hat{G}_{[N-2,0]} \\
\hat{G}_{[N-1,0]} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
\vdots \\
0 \\
0 \\
\end{bmatrix}
\]

(9) 

The matrix \( \hat{G}[k1,k2] \) can be calculated by the following equation:

\[
\begin{bmatrix}
\hat{G}_{[0,0]} & \hat{G}_{[0,1]} & \ldots & \hat{G}_{[N-1,N-2]} & \hat{G}_{[N-1,N-1]} \\
\hat{G}_{[1,0]} & \hat{G}_{[1,1]} & \ldots & \hat{G}_{[N-1,N-2]} & \hat{G}_{[N-1,N-1]} \\
\vdots & \vdots & \ldots & \vdots & \vdots \\
\hat{G}_{[N-2,0]} & \hat{G}_{[N-2,1]} & \ldots & \hat{G}_{[N-1,N-2]} & \hat{G}_{[N-1,N-1]} \\
\hat{G}_{[N-1,0]} & \hat{G}_{[N-1,1]} & \ldots & \hat{G}_{[N-1,N-2]} & \hat{G}_{[N-1,N-1]} \\
\end{bmatrix} \begin{bmatrix}
\gamma_{[0,0]} \\
\gamma_{[0,1]} \\
\vdots \\
\gamma_{[N-1,N-2]} \\
\gamma_{[N-1,N-1]} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
\vdots \\
0 \\
0 \\
\end{bmatrix}
\]

(10) 

When the matrix \( \hat{G}[k1,k2] \) is estimated, the energy factor \( G[k] \) can be calculated from the matrix \( \hat{G}[k1,k2] \). The relative equation is defined as below:

\[
G[k] = \sum_{n=0}^{L-1} e^{2\pi j \frac{n k W}{N} + f c - 2W} \gamma_{k} e^{j\theta_{k0}}^{n} 
\]

(11) 

In order to obtain the energy factor \( G[k] \), the complex equation \( G[k] = \gamma_{k} e^{j\theta_{k0}}^{n} \) is set. Thus, the above equation can be rewritten as below:

\[
\begin{bmatrix}
G_{[0,0]} & G_{[0,1]} & \ldots & G_{[0,N-2]} & G_{[0,N-1]} \\
G_{[1,0]} & G_{[1,1]} & \ldots & G_{[1,N-2]} & G_{[1,N-1]} \\
\vdots & \vdots & \ldots & \vdots & \vdots \\
G_{[N-2,0]} & G_{[N-2,1]} & \ldots & G_{[N-2,N-2]} & G_{[N-2,N-1]} \\
G_{[N-1,0]} & G_{[N-1,1]} & \ldots & G_{[N-1,N-2]} & G_{[N-1,N-1]} \\
\end{bmatrix} = \begin{bmatrix}
\gamma_{[0,0]} \\
\gamma_{[0,1]} \\
\vdots \\
\gamma_{[N-1,N-2]} \\
\gamma_{[N-1,N-1]} \\
\end{bmatrix}
\]

(12) 

In the above equation, the energy matrix \( \hat{G}[k1,k2] \) is known and can be estimated from Eq.11. However, the number of solution of \( \theta_{k} \) is more than one. Thus, the condition of \( \theta_{0} = 0 \) is set in the Eq.12. Then, the energy factor \( G[k] \) can be solved successfully. Moreover, in real case, the \( \cos_{f_{00}}[n] \) and \( \sin_{f_{00}}[n] \) defined in Eq.2 can be re-written as below:

\[
\cos_{f_{00}}[n] = \sum_{t=0}^{M-1} \sin(\frac{2\pi n k W}{SR} \left(\frac{4kW}{M} + f c - 2W\right)) * \gamma_{k} e^{j\theta_{k0}}^{n} 
\]

\[
\sin_{f_{00}}[n] = \sum_{t=0}^{M-1} \sin(\frac{2\pi n k W}{SR} \left(\frac{4kW}{M} + f c - 2W\right)) * \gamma_{k} e^{j\theta_{k0}}^{n} 
\]

(13) 

Some steps for our single detection algorithm can be summarized as below:

1. Set sample rate \( SR \) Hz/Sec.
2. Set frame length \( L \) of correlation function
3. Set the frequency \( f_{00} \) of single tone
4. Set the bandwidth \( W \) of band-active function
5. Estimate \( \gamma_{k}, \theta_{k} \) from Eq.12
6. Calculate \( \cos_{f_{00}}[k], \sin_{f_{00}}[k] \) from Eq.13
7. Calculate $y(f_0)$ from Eq.1
8. Make decision for the single tone detection

III. EXPERIMENTAL RESULTP

The experiment result of DTMF detection is employed to examine the performance of our method. The computation requirement of the single tone detection for each method is listed as Table 1. (Sample Rate=8000Hz, Frame Length=N sample points)

Table 1: The Computation Requirement for Each Method.

<table>
<thead>
<tr>
<th>Method</th>
<th>Multiply</th>
<th>Adder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goertzel Algorithm</td>
<td>2N+2</td>
<td>4N-2</td>
</tr>
<tr>
<td>MCA-QCS</td>
<td>2N+2</td>
<td>2N-1</td>
</tr>
<tr>
<td>SCA-QCS</td>
<td>9N</td>
<td>8N</td>
</tr>
</tbody>
</table>

The frequency response of the SCA-QCS with frame length of N=400 is depicted in Fig.1. The SCA-QCS and the Goertzel algorithm have the same frequency response. The bandwidth of [-2.5%, 2.5%] and the carrier-frequency with 852Hz are set. The bandwidth is not enough for the DTMF detection. In the Fig.2, the MCA-QCS with frame length of N=400 is depicted. The bandwidth and carrier frequency is same as in Fig.1. The frequency response is superior to Fig.1. The performance of MCA-QCS for frequency response is obviously good than SCA-QCS. To confirm the superior performance, more experiments will be analyzed and compared with other method in future.

IV. CONCLUSION

The MCA-QCS method for single tone detection is proposed in this article. The computation requirement is dramatic low and the performance is good than the Goertzel algorithm. The parameter of narrow-band discriminator can be easily adjusted and any specification for single tone detection can be easily achieved. This method can be widely employed on the DTMF detection and the goal cost-down can be easily achieved.

REFERENCES