Microwave Filters (8)

Periodic Structures

where the ABCD matrix represents the cascade of a transmission line section of length $\frac{d}{2}$, a shunt susceptance $b$, and another transmission line section of length $\frac{d}{2}$.

$$
\begin{bmatrix}
V_n \\
I_n
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix}
$$

where the ABCD matrix is:

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} =
\begin{bmatrix}
\cos \frac{\theta}{2} & j\sin \frac{\theta}{2} \\
-j\sin \frac{\theta}{2} & \cos \frac{\theta}{2}
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \frac{\theta}{2} & j\sin \frac{\theta}{2} \\
-j\sin \frac{\theta}{2} & \cos \frac{\theta}{2}
\end{bmatrix}
$$

$$
= 
\begin{bmatrix}
\cos \theta & \frac{b}{2} \sin \theta \\
\frac{b}{2} \cos \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\cos \theta & \frac{b}{2} \sin \theta \\
\frac{b}{2} \cos \theta & \cos \theta
\end{bmatrix}
$$

where $\theta = \beta_0 d$. 

Assume the port voltages and currents satisfy wave equation
\[ V_{n+1} = V_n e^{-\gamma d} \]
\[ I_{n+1} = I_n e^{-\gamma d} \]

Then,
\[
\begin{bmatrix}
V_n \\
I_n
\end{bmatrix}
= \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix}
= \begin{bmatrix}
V_{n+1} e^{\gamma d} \\
I_{n+1} e^{\gamma d}
\end{bmatrix}
\]
or
\[
\begin{bmatrix} A-e^{\gamma d} & B \\ C & D-e^{\gamma d} \end{bmatrix}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix}
= \begin{bmatrix} 0 \\
0
\end{bmatrix}
\]

For a nontrivial solution to exist, the following must be satisfied
\[
\det\begin{bmatrix} A-e^{\gamma d} & B \\ C & D-e^{\gamma d} \end{bmatrix} = 0
\]
That is
\[ AD + e^{2\gamma d} - (A+D)e^{\gamma d} - BC = 0. \]
The characteristic impedance of this wave is defined as
\[ Z_B = z_0 \frac{V_{n+1}}{I_{n+1}} = -\frac{BZ_0}{A-e^{\gamma d}} \]
Also
\[ e^{\gamma d} = \frac{A+D \pm \sqrt{(A+D)^2 - 4}}{2} \]
Then
\[ Z_B^\pm = \frac{-2BZ_0}{2A-A-D \mp \sqrt{(A+D)^2 - 4}} \]
For symmetrical unit cells, $A=D$, therefore

$$Z^\pm_B = \frac{\pm BZ_0}{\sqrt{A^2 - 1}}$$

The $\pm$ solutions correspond to the positively and negatively traveling waves, respectively.

Due to reciprocity, $AD-BC=1$, the above equation becomes

$$\cosh \gamma d = \frac{A+D}{2} = \cos \theta - \frac{b}{2} \sin \theta.$$ 

Let $\gamma = \alpha + j\beta$, then

$$\cosh \gamma d = \cosh \alpha d \cos \beta d + j \sinh \alpha d \sin \beta d = \cos \theta - \frac{b}{2} \sin \theta$$

1. $\sinh \alpha d = 0 \Rightarrow \alpha = 0$.
   a. Nonattenuating, propagating wave.
   b. Passband.
   c. $\cos \beta d = \cos \theta - \frac{b}{2} \sin \theta \Rightarrow \beta = \frac{\cos^{-1}\left(\cos \theta - \frac{b}{2} \sin \theta\right) + 2n\pi}{d}$
   d. $Z_B$ is pure real.

2. $\sinh \beta d = 0 \Rightarrow \beta d = 0, \pi, 2\pi, \ldots$
   a. Attenuating, nonpropagating wave.
   b. Stopband.
   c. $\pm \cosh \alpha d = \cos \theta - \frac{b}{2} \sin \theta$
Has two solutions $\pm \alpha$ only when $|\cos \theta - \frac{b}{2}
abla \theta| \geq 1$

\[ d. \quad Z_B \text{ is pure imaginary.} \]

**Terminated Periodic Structures**

\[ V_N = V_N^+ + V_N^- = Z_L I_N = Z_L \left( \frac{V_N^+}{Z_B^+} + \frac{V_N^-}{Z_B^-} \right) \]

\[ \therefore \quad \Gamma = \frac{V_N^-}{Z_L/Z_B^- - 1} = \frac{Z_L - Z_B}{Z_L + Z_B} \]

if $Z_B = Z_B^+ = -Z_B^-$. 

**Example 8.1**

$Z_0 = 50 \Omega$, $d = 1.0 \text{ cm}$, $C_0 = 2.666 \text{ pF}$, $f = 3.0 \text{ GHz}$, $k = k_0$

$\nu_p = 0.42c$, slow wave.
Filter Design by The Image Parameter Method (8.2)

$Z_{i1}$ = input impedance at port 1 when port is terminated with $Z_{i2}$.

$Z_{i2}$ = input impedance at port 1 when port is terminated with $Z_{i1}$.

From ABCD matrix, we have

\[
Z_{ii} = \frac{V_1}{I_1} = \frac{AV_1 + BI_1}{CV_1 + DI_1} = \frac{AZ_{i2} + B}{CZ_{i2} + D}
\]

\[
Z_{i2} = \frac{V_2}{I_2} = \frac{DV_1 - BI_1}{-CV_1 + AI_1} = \frac{DZ_{i1} + B}{CZ_{i1} + A}
\]

Solving $Z_{i1}$ and $Z_{i2}$ gives

\[
Z_{i1} = \sqrt{\frac{AB}{CD}}
\]

\[
Z_{i2} = \sqrt{\frac{BD}{AC}}
\]

and $Z_{i2} = \frac{DZ_{i1}}{A}$. If symmetric, $A = D$ and $Z_{i2} = Z_{i1}$.

If port 2 is terminated in $Z_{i2}$, Voltage ratio equals

\[
\frac{V_2}{V_1} = D - \frac{B}{Z_{i1}} = D - B \sqrt{\frac{CD}{AB}} = \sqrt{\frac{D}{A} (\sqrt{AD} - \sqrt{BC})}
\]
Current ratio
\[ \frac{I_2}{I_1} = -C \frac{V_1}{I_1} + A = -CZ_{in} + A = \sqrt{\frac{AD}{D}}(\sqrt{AD} - \sqrt{BC}) \]

Define propagation factor as
\[ e^{-\gamma} = e^{-(\alpha + j\beta)} = \sqrt{AD} - \sqrt{BC} \]

or
\[ \cosh\gamma = \sqrt{AD} \]

**Constant-k Filter Sections**

Low pass filters

From Table 8.1, the image impedance equals
\[ Z_{iT} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_c^2}} \]

where
\[ \omega_c = \frac{2}{\sqrt{LC}}, \quad R_0 = \sqrt{\frac{L}{C}} = k \]

and
\[ e^\gamma = \frac{2\omega^2}{\omega_c^2} \left( 1 - \frac{2\omega^2}{\omega_c^2} \right) + \frac{2\omega}{\omega_c} \sqrt{\frac{\omega^2}{\omega_c^2} - 1} \]

1. \( \omega < \omega_c \): Passband. \( z_{it} \) is real. \( \gamma \) is imaginary. \( |e^\gamma| = 1 \).
2. \( \omega > \omega_c \): Stopband. \( z_{it} \) is imaginary. \( \gamma \) is real. \( -1 < e^\gamma < 0 \). Attenuation rate for \( \omega \rightarrow \omega_c \) is 40 dB/decade.

Disadvantages:
1. Slow cutoff.
2. Image impedance is a function of frequency.

High Pass Filters

\[
\begin{align*}
\omega_c &= \frac{1}{2\sqrt{LC}}, & R_0 &= \sqrt{\frac{L}{C}}
\end{align*}
\]
### TABLE 8.1 Image Parameters for T- and π-Networks

<table>
<thead>
<tr>
<th>T-Network</th>
<th>π-Network</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="T-Network Diagram" /></td>
<td><img src="image2.png" alt="π-Network Diagram" /></td>
</tr>
</tbody>
</table>

**ABCD parameters:**

<table>
<thead>
<tr>
<th>T-Network</th>
<th>π-Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 1 + Z_1/Z_2</td>
<td>A = 1 + Z_1/2Z_2</td>
</tr>
<tr>
<td>B = Z_1 + Z_1/Z_2</td>
<td>B = Z_1</td>
</tr>
<tr>
<td>C = 1/Z_2</td>
<td>C = 1/Z_2 + Z_1/4Z_2</td>
</tr>
<tr>
<td>D = 1 + Z_1/Z_2</td>
<td>D = 1 + Z_1/2Z_2</td>
</tr>
</tbody>
</table>

**Z parameters:**

<table>
<thead>
<tr>
<th>T-Network</th>
<th>π-Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_{11} = Z_{22} = Z_2 + Z_1/2</td>
<td>Y_{11} = Y_{22} = 1/Z_1 + 1/2Z_2</td>
</tr>
<tr>
<td>Z_{12} = Z_{21} = Z_2</td>
<td>Y_{12} = Y_{21} = 1/Z_1</td>
</tr>
</tbody>
</table>

**Image impedance:**

<table>
<thead>
<tr>
<th>T-Network</th>
<th>π-Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_{IT} = \sqrt{Z_1Z_2(1 + 1/4Z_2)}</td>
<td>Z_{Iπ} = \sqrt{Z_1Z_2/1 + Z_1/4Z_2} = Z_1Z_2/Z_{IT}</td>
</tr>
</tbody>
</table>

**Propagation constant:**

<table>
<thead>
<tr>
<th>T-Network</th>
<th>π-Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>e^\alpha = 1 + Z_1/2Z_2 + \sqrt{Z_1/Z_2 + Z_1/Z_2}</td>
<td>e^\beta = 1 + Z_1/2Z_2 + \sqrt{Z_1/Z_2 + Z_1/Z_2}</td>
</tr>
</tbody>
</table>

---

### Figure 8.1

Passband | Stopband
---|---
\(\omega_c\) | \(\omega\)

---

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m-Derived Filter Sections

Let 

\[ Z'_1 = mZ_1 \]

In order to keep the image impedance the same

\[ Z_{im} = \sqrt{Z_1Z_2 + \frac{Z_1^2}{4}} = \sqrt{Z'_1Z'_2 + \frac{Z'_1^2}{4}} = \sqrt{mZ_1Z'_2 + \frac{m^2Z_1^2}{4}} \]

\[ \therefore Z'_2 = \frac{Z_2}{m} + \frac{1-m^2}{4m} Z_1 \]

For a low pass filter, \( Z_1 = j\omega L \) and \( Z_2 = \frac{1}{j\omega C} \). Then

\[ Z'_1 = j\omega Lm \]

\[ Z'_2 = \frac{1}{j\omega Cm} + \frac{1-m^2}{4m} j\omega L \]

\[ e^\gamma = \sqrt{1 + \frac{Z'_1}{2Z'_2} + \frac{Z'_1(1 + \frac{Z'_1}{4Z'_2})}{Z'_2}} \]

where
\[
\frac{Z_1'}{Z_2'} = \frac{-2(m\omega/\omega_c)^2}{1 - (1-m^2)(\omega/\omega_c)^2}
\]

\[
1 + \frac{Z_1'}{4Z_2'} = \frac{1 - (\omega/\omega_c)^2}{1 - (1-m^2)(\omega/\omega_c)^2}
\]

If \(0 < m < 1\), \(e^\gamma\) is real, and \(|e^\gamma| > 1\) for \(\omega > \omega_c\). Stopband.

When \(\omega = \frac{\omega_c}{\sqrt{1-m^2}}\), \(|e^\gamma| = \infty\), implying infinite attenuation.

Disadvantages:
1. Slow attenuation when \(\omega \to \infty\).
2. Image impedance is a function of frequency.
1. p. 434, in Table 8.1, the propagation constant should be \( e^\gamma = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2}} \).

2. p. 434, Eq. 8.36, add absolute value to the term \( 1 - \frac{2\omega^2}{\omega_c^2} \).

3. p. 437, Eq. 8.43, add absolute value to the term \( 1 + \frac{Z_1'}{2Z_2'} \).

### TABLE 8.2 Summary of Composite Filter Design

<table>
<thead>
<tr>
<th>Low-Pass</th>
<th>High-Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant-(L) T section</strong></td>
<td><strong>Constant-(L) T section</strong></td>
</tr>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>( R_0 = \sqrt{L/C} )</td>
<td>( R_0 = \sqrt{L/C} )</td>
</tr>
<tr>
<td>( v_0 = 2\sqrt{L/C} )</td>
<td>( v_0 = 2\sqrt{L/C} )</td>
</tr>
<tr>
<td>( C = 2v_0R_0 )</td>
<td>( C = 2v_0R_0 )</td>
</tr>
<tr>
<td><strong>(m)-derived (T) section</strong></td>
<td><strong>(m)-derived (T) section</strong></td>
</tr>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>( m = \sqrt{1 - \epsilon_0^2/\epsilon_m^2} ) for sharp cutoff</td>
<td>( m = \sqrt{1 - \epsilon_0^2/\epsilon_m^2} ) for sharp cutoff</td>
</tr>
<tr>
<td>( 0.6 ) for matching</td>
<td>( 0.6 ) for matching</td>
</tr>
<tr>
<td><strong>Bisected-(\pi) matching section</strong></td>
<td><strong>Bisected-(\pi) matching section</strong></td>
</tr>
<tr>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>( R_0 )</td>
<td>( R_0 )</td>
</tr>
<tr>
<td>( 2\sqrt{m} )</td>
<td>( 2\sqrt{m} )</td>
</tr>
<tr>
<td>( 2L/m )</td>
<td>( 2L/m )</td>
</tr>
<tr>
<td>( Z_{pi} )</td>
<td>( Z_{pi} )</td>
</tr>
</tbody>
</table>

Table 8.2
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**Goals:**

1. Provide the same form of the propagation factor of an m-derived T-section.
2. Provide a less frequency dependent image impedance.

**Procedures:**

1. Use Table 8.1, to find out the equivalent [\(\Pi\)]- section that give the same the propagation constants as the T-section.
2. Compute the image impedance.

\[
Z_{im} = \frac{Z_1^2}{Z_{IT}} = \frac{Z_1 Z_2 + Z_1^2 (1-m^2)/4}{1-(1-m^2)(\omega/\omega_c)^2} \frac{1}{R_0 \sqrt{1-(\omega/\omega_c)^2}} R_0
\]

Choose \(m=0.6\) to minimize the variation.
3. Now \( Z_{i\pi} \neq Z_{iT} \). Split the \( \Pi \)-section to two parts to match \( Z_{iT} \). It can be proved that the image impedances equals to \( Z_{iT} \) and \( Z_{i\pi} \) respectively.

4. Let the propagation constant of the original \( \Pi \)-section be \( e^\gamma \) and the split \( \Pi \)-section \( e^{\gamma_1}, e^{\gamma_2} \). It is obviously \( e^\gamma = e^{\gamma_1} e^{\gamma_2} \).

**Composite Filters**

1. From the given impedance \( R_0 \) and the cut-off frequency \( \omega_c \), determine the values of the inductance \( L \) and the capacitance \( C \) by

\[
R_0 = \sqrt{\frac{L}{C}}, \quad \omega_c = \frac{2}{\sqrt{LC}}.
\]

Then, the constant-k T section can be implemented.

2. Determine the m value from the infinite attenuation pole by

\[
m = \sqrt{1 - \left( \frac{\omega_c}{\omega_o} \right)^2}
\]
Then m-derived T section can be implemented.

3. Choose $m=0.6$ as the m value of the bisected $\Pi$-section.

Example 8.2: LOW-PASS COMPOSITE FILTER DESIGN

$Z_0=75\,\Omega$, $f_c=2\,\text{MHz}$, $f_\infty=2.05\,\text{MHz}$. 
From Table 8.2

1. Constant-k T section

\[ L = \frac{2R_0}{\omega_c} = 11.94 \mu H, \quad C = \frac{2}{R_0 \omega_c} = 2.122 \text{nF} \]

2. \( m \)-derived T section

\[ m = \sqrt{1 - \left( \frac{f_c}{f_\infty} \right)^2} = 0.2195 \]

\[ \frac{mL}{2} = 1.310 \mu H, \quad mC = 465.8 \text{ pF}, \quad \frac{1 - m^2}{4m} L = 12.94 \mu H \]

3. \( m=0.6 \) matching section

\[ \frac{mL}{2} = 3.582 \mu H, \quad mC = 636.5 \text{ pF}, \quad \frac{1 - m^2}{4m} L = 6.368 \mu H \]
Filter Design by the Insertion Loss Method (8.3)

Benefits: can synthesize a desired response systematically. Disadvantage: trade-off between the ideal and real responses.

Define: insertion loss or power loss ratio, $P_{LR}$ by

$$ P_{LR} = \frac{\text{Power available source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{load}} = \frac{1}{1 - |\Gamma(\omega)|^2} $$

Since $|\Gamma(\omega)|^2$ is an even function,

$$ |\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)} \Rightarrow P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)} $$

Maximally flat:

$$ P_{LR} = 1 + k^2 \left( \frac{\omega}{\omega_c} \right)^{2N} $$

1. Order: $N$
2. Low pass, binomial or Butterworth response.
3. Flattest passband.
4. Cutoff frequency: $\omega_c$
5. Passband: $0 \leq \omega_c$.
6. Power loss ratio at $\omega_c$: $1 + k^2$. For $k=1$, 3-dB loss.
7. When $\omega \geq \omega_c$, $P_{LR} = k^2 \left( \frac{\omega}{\omega_c} \right)^{2N}$.
8. Attenuation rate: $20N$ dB/decade
9. The first $(2N-1)$ derivatives are zero at $\omega=0$

Equal ripple:

$$ P_{LR} = 1 + k^2 T_N^2 \left( \frac{\omega}{\omega_c} \right) $$

1. Order: $N$
2. Low pass.
3. $T_N$ is the Chebyshev polynomial of order $N$. 
4. Equal ripple of amplitude $1+k^2$.
5. When $\omega \gg \omega_c$, $P_{LR} \approx \frac{k^2}{4} \left( \frac{2\omega}{\omega_c} \right)^{2N}$.
6. Sharper cutoff than maximally flat types by a factor $\frac{2^{2N}}{4}$.

**Linear Phase:**

$$\phi(\omega) = A\omega \left[ 1 + p \left( \frac{\omega}{\omega_c} \right)^{2N} \right]$$

Then the group delay

$$\tau_d = \frac{d\phi}{d\omega} = A \left[ 1 + p(2N+1) \left( \frac{\omega}{\omega_c} \right)^{2N} \right]$$

is maximally flat.

**FIGURE 8.21** Maximally flat and equal-ripple low-pass filter responses ($N =$ ?)
Maximally Flat Low-Pass Filter Prototype

Consider the above circuit,
\[ Z_{in} = j\omega L + \frac{R(1-j\omega RC)}{1+\omega^2 R^2 C^2} \]

Since
\[ \Gamma = \frac{Z_{in} - 1}{Z_{in} + 1} \]

We have
\[ P_{LR} = \frac{1}{1 - |\Gamma|^2} = 1 + \frac{1}{4R} [(1-R)^2 + (R^2 C^2 + L^2 - 2LC R^2) \omega^2 + L^2 C^2 R^2 \omega^4] \]

In order to fit the above equation to a maximally flat low-pass filter with \( \omega_c = 1, k = 1, \) and \( N = 2, \) that is,
\[ P_{LR} = 1 + \omega^4, \]

It is required that
\[ 1-R = 0, \]
\[ R^2 C^2 + L^2 - 2LC R^2 = 0, \text{ and} \]
\[ \frac{L^2 C^2 R^2}{4R} = 1. \]

Solving for \( R, L \) and \( C, \) we have
$R=1$, $L=C=\sqrt{2}$.

Procedures:
1. Referring to Fig. 8.26, determine the order from the required attenuation characteristics.
2. By looking up Table 8.3, determine the normalized component values by the following rules:
   a. $g_0$ is the generator resistance if the ladder circuit starts with a shunt capacitance, or generator conductance if starts with a series inductors.
   b. $g_k$ is the inductance for series inductors, or capacitance for shunt capacitors.
   c. $g_{N+1}$ the load resistance if $g_N$ is a shunt capacitor, or load conductance if $g_N$ is a series inductor.
3. Impedance and frequency scaling. Let $R_0$ and $\omega_c$ be the real generator resistance and cutoff frequency respectively, then
Example 8.3
Design a maximal flat low-pass filter with a cutoff frequency of 2 GHz, impedance of 50Ω, and at least 15 dB insertion loss at 3 GHz

\[
\frac{\omega}{\omega_c} - 1 = 0.5
\]

\[g_1 = 0.618 = g_5, \quad g_2 = 1.618 = g_4, \quad g_3 = 2.\]

\[C_1' = 0.984 \text{ pF} = C_5', \quad L_2' = 6.438 \text{ nH} = L_4', \quad C_3' = 3.183 \text{ pF}.\]
Figure 8.30
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Equal Ripple Low-Pass Filter Prototype

Procedures:
1. Referring to Fig. 8.27, determine the order from the required attenuation characteristics and choose the desired ripple level.
2. Decide the component values by Table 8.4 and follow the same procedure in the maximal flat prototype. Note when N is even, the generator resistance and the load resistance are not equal.
3. Impedance and frequency scaling. Use the same procedure in the maximal flat prototype.
FIGURE 8.27  Attenuation versus normalized frequency for equal-ripple filter prototypes. (a) 0.5 dB ripple level. (b) 3.0 dB ripple level.
Linear Phase Low-Pass Filter Prototypes
Procedure: Same as previous prototypes except using Table 8.5.

Filter Transformation (8.4)

Goals:
1. Low pass $\rightarrow$ high pass.
2. Low pass $\rightarrow$ band pass.
3. Low pass $\rightarrow$ band stop.

Low Pass $\rightarrow$ High Pass

Let $\omega \rightarrow -\frac{\omega_c}{\omega}$, then the low pass response becomes a high pass
response.

Procedure:
1. Convert the inductances in the low pass prototypes to capacitances as follows
   \[ jX_k = j\omega g_k = -j \frac{\omega_c}{\omega} g_k \Rightarrow C'_k = \frac{1}{j\omega C_k} \Rightarrow C'_k = \frac{1}{\omega_c g_k} \]
2. Convert the capacitances in the low pass prototypes to inductances as follows
   \[ jB_k = j\omega g_k = -j \frac{\omega_c}{\omega} g_k \Rightarrow L'_k = \frac{1}{j\omega L_k} \Rightarrow L'_k = \frac{1}{\omega_c g_k} \]
3. Include the effect of impedance scaling
   \[ C'_k = \frac{1}{\omega_c g_k R_0}, \quad L'_k = \frac{R_0}{\omega_c g_k} \]

**Low Pass → Band Pass**

Let \( \omega \rightarrow \frac{1}{\Delta} \left( \frac{\omega'}{\omega_0} - \frac{\omega_0}{\omega'} \right) \), where \( \Delta = \frac{\omega_2 - \omega_0}{\omega_0} > 0 \) and \( \omega_0 = \sqrt{\omega_1 \omega_2} \). Then,

\[
\begin{align*}
\omega' &= \omega_0 \Rightarrow \frac{1}{\Delta} \left( \frac{\omega'}{\omega_0} - \frac{\omega_0}{\omega'} \right) = 0 \\
\omega' &= \omega_1 \Rightarrow \frac{1}{\Delta} \left( \frac{\omega'}{\omega_0} - \frac{\omega_0}{\omega'} \right) = -1 = -\omega_c \\
\omega' &= \omega_2 \Rightarrow \frac{1}{\Delta} \left( \frac{\omega'}{\omega_0} - \frac{\omega_0}{\omega'} \right) = 1 = \omega_c
\end{align*}
\]

which is a band pass response with cutoff at \( \omega_1 \) and \( \omega_2 \).

An inductance in the low pass prototype will converts to a series inductance and a series capacitance as follow
An capacitance in the low pass prototype will converts to a parallel inductance and a parallel capacitance as follow

\[ jX_k = j\omega g_k = j \frac{1}{\Delta} \left( \frac{\omega'}{\omega_0} - \frac{\omega_0}{\omega'} \right) g_k = j \frac{\omega' g_k}{\Delta \omega_0} - j \frac{\omega_0 g_k}{\Delta \omega'} \Rightarrow L'_k = \frac{g_k}{\Delta \omega_0}, \quad C'_k = \frac{\Delta}{\omega_0 g_k} \]

Notice that both pairs of inductances and capacitance have the same resonant frequency \( \omega_0 \).

**Low Pass → Band Stop**

Similarly, Let \( \omega \rightarrow \Delta \left( \frac{\omega'}{\omega_0} - \frac{\omega_0}{\omega'} \right)^{-1} \).

Convert an inductance in the low pass prototype to a parallel inductance and a parallel capacitance as follow

\[ L'_k = \frac{\Delta g_k}{\omega_0}, \quad C'_k = \frac{1}{\Delta \omega_0 g_k} \]

Convert an capacitance in the low pass prototype to a series inductance and a series capacitance as follow

\[ C'_k = \frac{\Delta g_k}{\omega_0}, \quad L'_k = \frac{1}{\Delta \omega_0 g_k} \]
To sum up

![Graphs showing different filter responses with captions (a), (b), and (c).]

**TABLE 8.6  Summary of Prototype Filter Transformations** \( \Delta = \frac{\omega_2 - \omega_1}{\omega_0} \)

<table>
<thead>
<tr>
<th>Low-pass</th>
<th>High-pass</th>
<th>Bandpass</th>
<th>Bandstop</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Low-pass diagram]</td>
<td>![High-pass diagram]</td>
<td>![Bandpass diagram]</td>
<td>![Bandstop diagram]</td>
</tr>
</tbody>
</table>

\[ \frac{1}{\omega_0 C} \quad \frac{\Delta}{\omega_0 C} \quad \frac{C}{\omega_0 \Delta} \quad \frac{1}{\omega_0 L \Delta} \]

Table 8.6

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Example 8.4: BANDPASS FILTER DESIGN

N=3, 0.5 dB equal ripple, \( f_0 = 1 \text{ GHz} \), \( \frac{\Delta f}{f} = 10\% \), \( Z_0 = 50 \Omega \).

\[ L_1' = 127 \text{nH} \]
\[ C_1' = 0.199 \text{pF} \]
\[ L_2' = 0.726 \text{nH} \]
\[ C_2' = 34.91 \text{pF} \]
\[ L_3' = 127 \text{nH} \]
\[ C_3' = 0.199 \text{pF} \]
Filter Implementation (8.5)

Richard’s Transformation

\[ jX_L = jL\tan\beta\ell \]
\[ jB_C = jC\tan\beta\ell \]

Choose \( \ell = \lambda/8 \) at \( \omega_c (= 1) \) such that \( jX_L = jL \) and \( jX_L = jC \).

A zero occur at \( 2\omega_c(\ell = \lambda/4) \).

Kuroda’s identities

- Physically separate transmission line stubs.
- Transform series stubs into shunt stubs, or vice versa.
- Change impractical characteristic impedance into more realizable ones.
Consider Table 8.7(a),

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_L = \begin{bmatrix}
1 & 0 \\
\frac{j\Omega}{Z_2} & 1
\end{bmatrix} \begin{bmatrix}
1 & j\Omega Z_1 \\
\frac{j\Omega}{Z_1} & 1
\end{bmatrix} \frac{1}{\sqrt{1+\Omega^2}}
\]

\[
= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix}
1 & j\Omega Z_1 \\
-j\Omega \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right) & 1 - \Omega^2 \frac{Z_1}{Z_2}
\end{bmatrix}
\]

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_R = \begin{bmatrix}
1 & j\Omega \frac{Z_2}{n^2} \\
\frac{\Omega Z_2}{n^2} & 1
\end{bmatrix} \begin{bmatrix}
1 & j\Omega \frac{Z_1}{n^2} \\
0 & 1
\end{bmatrix} \frac{1}{\sqrt{1+\Omega^2}}
\]

\[
= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix}
1 & j\Omega \frac{n^2 (Z_1 + Z_2)}{Z_2} \\
-j\Omega \frac{n^2 Z_1}{Z_2} & 1 - \Omega^2 \frac{Z_1}{Z_2}
\end{bmatrix}
\]
where \( \Omega = \tan \beta l \).

Since the left and the right ABCD matrix must be the same, we have

\[
n^2 = 1 + \frac{Z_2}{Z_1}\]

Example 8.5 LOW-PASS FILTER DESIGN USING STUBS

\( f_c = 4 \text{ GHz}, \ Z_0 = 50 \Omega, \ N = 3, \) 3 dB equal ripple.
Impedance and Admittance Inverters

**Impedance Inverters**

- $Z_{in} = K^2 Z_o$
- $Z_o = K$
- $Z_o = \frac{V_o}{I}$

**Admittance Inverters**

- $Y_{in} = J^2 Y_L$
- $Y_L = J$
- $Y_L = \frac{I}{V_o}$

---

Figure 8.3(a) and 8.3(b) from Microwave Circuits, 32nd edition, John Wiley & Sons, Inc. All rights reserved.
Stepped-Impedance Low-Pass Filters (8.6)

Convert the ABCD matrix of a short transmission line to Z-parameters, we have

\[ Z_{11} = Z_{22} = \frac{A}{C} = -jZ_0 \cot \beta \ell \]

\[ Z_{12} = Z_{21} = \frac{1}{C} = -jZ_0 \csc \beta \ell \]

Using T equivalent circuit, we have

\[ \frac{X}{2} = Z_0 \tan \frac{\beta \ell}{2} \]

\[ B = \frac{1}{Z_0} \sin \beta \ell \]

If \( Z_0 \) is large

\[ X \approx Z_0 \beta \ell \]

\[ B \approx 0 \]

If \( Z_0 \) is small

\[ X \approx 0 \]

\[ B \approx Y_0 \beta \ell \]

To sum up,
Inductor: $\beta = \frac{LR_0}{Z_h}$

Capacitor: $\beta = \frac{CZ_0}{R_0}$

Example 8.7 STEPPED IMPEDANCE FILTER DESIGN

$f_c = 2.5 \text{GHz}$, $Z_0 = 50$, 20 dB attenuation at 4 GHz. Maximal flat.

$Z_h = 120 \Omega$, $Z_0 = 20 \Omega$. Substrate: $d = 0.158 \text{cm}$, $\varepsilon_r = 4.2$, $\tan\delta = 0.02$. 

![Diagram of stepped impedance filter design](image)
For a open-circuit transmission line,
\[ Z = -jZ_{0n} \cot \theta, \quad \theta = \beta \ell \]

where \( \theta = \frac{\pi}{2} \) for \( \omega = \omega_0 \). Let \( \omega = \omega_0 + \Delta \omega \), where \( \Delta \omega \approx \omega_0 \). Assume an ideal transmission line, then \( \theta = \frac{\pi}{2} (1 + \frac{\Delta \omega}{\omega_0}) \). Now

\[ Z = -jZ_{0n} \cot \left( \frac{\pi}{2} + \frac{\pi}{2} \frac{\Delta \omega}{\omega_0} \right) = jZ_{0n} \tan \frac{\pi}{2} \frac{\Delta \omega}{\omega_0} = jZ_{0n} \frac{\pi}{2} \frac{\omega - \omega_0}{\omega_0} \]

For a series LC circuit near resonance

\[ Z = j\omega L_n + \frac{1}{j\omega C_n} = j \sqrt{\frac{L_n}{C_n}} (\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}) = j \sqrt{\frac{L_n}{C_n}} \frac{\omega^2 - \omega_0^2}{\omega_0 \omega} = 2jL_n (\omega - \omega_0) \]

\[ \omega_0 = \frac{1}{\sqrt{L_n C_n}} \]

Thus
For the circuit

\[ Z_{0n} = \frac{4\omega_0 L_n}{\pi} \]
\[ Y = \frac{1}{j\omega L_2 + \frac{1}{j\omega C_2}} + \frac{1}{Z_0^2} \left[ \frac{1}{j\omega L_1 + \frac{1}{j\omega C_1}} + \frac{1}{Z_0} \right]^{-1} \]

\[ = \frac{1}{j\sqrt{L_2/C_2} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]} + \frac{1}{Z_0^2} \left\{ \frac{1}{j\sqrt{L_1/C_1} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]} + \frac{1}{Z_0} \right\}^{-1} \]

Which should equal to the bandstop prototype

\[ Y = \frac{1}{j\omega L_2'} + \frac{1}{j\omega C_2'} \left[ \frac{1}{j\omega C_1' + \frac{1}{j\omega L_1'}} + Z_0 \right]^{-1} \]

\[ = \frac{1}{j\sqrt{L_2'/C_2'} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]} + \left\{ \frac{1}{j\sqrt{C_1'/L_1'} \left[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]} \right\}^{-1} \]

To make the two values the same, we need

\[ \frac{1}{Z_0^2} \sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{C_1'}{L_1'}} \sqrt{\frac{L_2}{C_2}} = \sqrt{\frac{L_2'}{C_2'}} \]

Solving for the above equations, we have

\[ L_1 = \frac{Z_0^2}{\omega_0^2 L_1'}, \quad L_2 = L_2' \]

Then,

\[ Z_{01} = \frac{4Z_0^2}{\pi \omega_0 L_1'}, \quad Z_{02} = \frac{4Z_0}{\pi g_1 \Delta}, \quad Z_{02} = \frac{4Z_0}{\pi g_2 \Delta} \]

where \( \Delta = \frac{\omega_2 - \omega_1}{\omega_0} \).

To sum up,

bandstop filter: open \( \frac{\lambda}{4} \) stub with \( Z_{0n} = \frac{4Z_0}{\pi g_n \Delta} \).
bandpass filter: short stub with $Z_{0n} = \frac{\pi Z_0 \Delta}{4g_n}$

Example 8.8 BANDSTOP FILTER DESIGN

$f_0 = 2$ GHz, $\frac{\Delta f}{f_0} = 10\%$, $Z_0 = 50\Omega$, $N=3$, 0.5 dB equal ripple.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$g_n$</th>
<th>$Z_{0n}$ ($\Omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5963</td>
<td>265.9</td>
</tr>
<tr>
<td>2</td>
<td>1.0967</td>
<td>387.0</td>
</tr>
<tr>
<td>3</td>
<td>1.5963</td>
<td>265.9</td>
</tr>
</tbody>
</table>
Coupled Line Filters

Even-Odd Mode Analysis
Denote the left side as port 1 and the right side port 2.
Let $V_1^e, I_1^e, V_1^o, I_1^o, V_2^e, I_2^e, V_2^o, I_2^o$ be the even-odd mode voltages and currents.
Goal: find out the impedance matrices
1. Even mode:
2. Odd mode:

\[
\begin{bmatrix}
V_1^e \\
V_2^e \\
V_1^o \\
V_2^o
\end{bmatrix} =
\begin{bmatrix}
Z_{11}^e & Z_{12}^e \\
Z_{21}^e & Z_{22}^e
\end{bmatrix}
\begin{bmatrix}
I_1^e \\
I_2^e \\
I_1^o \\
I_2^o
\end{bmatrix}
\Rightarrow
\begin{cases}
Z_{11}^e = \frac{V_1^e}{I_1^e} = \frac{V^+ e^{j\theta} + V^- e^{-j\theta}}{I^+ e^{j\theta} - I^- e^{-j\theta}} = -jZ_0 e^{j\theta} = Z_{22}^e \\
Z_{21}^e = \frac{V_2^e}{I_1^e} = \frac{2V^+}{I^+ e^{j\theta} - I^- e^{-j\theta}} = -jZ_0 e^{-j\theta} = Z_{12}^e
\end{cases}
\]

\[
\begin{bmatrix}
V_1^o \\
V_2^o \\
V_1^o \\
V_2^o
\end{bmatrix} =
\begin{bmatrix}
Z_{11}^o & Z_{12}^o \\
Z_{21}^o & Z_{22}^o
\end{bmatrix}
\begin{bmatrix}
I_1^o \\
I_2^o \\
I_1^o \\
I_2^o
\end{bmatrix}
\Rightarrow
\begin{cases}
Z_{11}^o = \frac{V_1^o}{I_1^o} = \frac{V^+ e^{j\theta} + V^- e^{-j\theta}}{I^+ e^{j\theta} - I^- e^{-j\theta}} = -jZ_0 e^{j\theta} = Z_{22}^o \\
Z_{21}^o = \frac{V_2^o}{I_1^o} = \frac{2V^+}{I^+ e^{j\theta} - I^- e^{-j\theta}} = -jZ_0 e^{-j\theta} = Z_{12}^o
\end{cases}
\]

\[\theta = \beta \ell\]

To sum up,

\[
\begin{bmatrix}
V_1^e \\
V_2^e \\
V_1^o \\
V_2^o
\end{bmatrix} =
\begin{bmatrix}
Z_{11}^e & Z_{12}^e & 0 & 0 \\
Z_{21}^e & Z_{22}^e & 0 & 0 \\
0 & 0 & Z_{11}^o & Z_{12}^o \\
0 & 0 & Z_{21}^o & Z_{22}^o
\end{bmatrix}
\begin{bmatrix}
I_1^e \\
I_2^e \\
I_1^o \\
I_2^o
\end{bmatrix}
\]

Since

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 1 & 0 \\
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 \\
0 & 1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
V_1^e \\
V_2^e \\
V_1^o \\
V_2^o
\end{bmatrix}
\]

We have

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 1 & 0 \\
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 \\
0 & 1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_1^e \\
I_2^e \\
I_1^o \\
I_2^o
\end{bmatrix}
\]

Let port 2 and 4 open, we have

\[
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 1 & 0 \\
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 \\
0 & 1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
Z_{11}^e & Z_{12}^e & 0 & 0 \\
Z_{21}^e & Z_{22}^e & 0 & 0 \\
0 & 0 & Z_{11}^o & Z_{12}^o \\
0 & 0 & Z_{21}^o & Z_{22}^o
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 1 & 0 \\
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 \\
0 & 1 & 0 & 1
\end{pmatrix}^{-1}
\begin{pmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4
\end{pmatrix}
\]

\[
\begin{pmatrix}
Z_{11}^e+Z_{11}^o \\
Z_{11}^e-Z_{11}^o \\
Z_{12}^e-Z_{12}^o \\
Z_{12}^e+Z_{12}^o
\end{pmatrix}
= \frac{2}{2}
\]

\[
\begin{pmatrix}
Z_{11}^e+Z_{11}^o \\
Z_{11}^e-Z_{11}^o \\
Z_{12}^e+Z_{12}^o \\
Z_{12}^e-Z_{12}^o
\end{pmatrix}
= \frac{2}{2}
\]

\[
\begin{pmatrix}
Z_{11}^e+Z_{11}^o \\
Z_{11}^e-Z_{11}^o \\
Z_{12}^e+Z_{12}^o \\
Z_{12}^e-Z_{12}^o
\end{pmatrix}
= \frac{2}{2}
\]

\[
\begin{pmatrix}
Z_{11}^e+Z_{11}^o \\
Z_{11}^e-Z_{11}^o \\
Z_{12}^e+Z_{12}^o \\
Z_{12}^e-Z_{12}^o
\end{pmatrix}
= \frac{2}{2}
\]

Computing the image impedance

\[
Z_i = \sqrt{Z_{11}^2 - \frac{Z_{11}^2 Z_{13}^2}{Z_{33}^2}} = \frac{1}{2} \sqrt{(Z_{0e} - Z_{0o})^2 \csc^2 \theta - (Z_{0e} + Z_{0o})^2 \cot^2 \theta}
\]

If \( \frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow Z_i = \frac{Z_{0e} - Z_{0o}}{2} \) which is real and positive since

\[Z_{0e} > Z_{0o} \]
\[ \theta \rightarrow 0, \pi \Rightarrow Z_i \rightarrow \pm j\infty. \]

Cutoff frequency

\[ \cos \theta_1 = -\cos \theta_2 = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \]

Propagation constant

\[ \cos \beta = \sqrt{\frac{Z_{11}Z_{33}}{Z_{13}^2}} = \frac{Z_{11}}{Z_{13}} = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \cos \theta \]

Passband

\[ \beta \text{ is real for } \theta_1 < \theta < \theta_2 = \pi - \theta_1 \]

The image impedance of the above is

\[ Z_i = \sqrt{\frac{JZ_0^2 \sin^2 \theta - (1/J) \cos^2 \theta}{(1/JZ_0^2) \sin^2 \theta - J \cos^2 \theta}} \]

At \( \theta = \frac{\pi}{2} \), \( Z_i = JZ_0^2 \). The propagation constant is

\[ \cos \beta = A = (JZ_0 + \frac{1}{JZ_0}) \sin \theta \cos \theta \]

Comparing to the original equations, we have

\[ \frac{1}{2} (Z_{0e} - Z_{0o}) = JZ_0^2 \]

\[ \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} = JZ_0^2 + \frac{1}{JZ_0} \]

with \( \sin \theta = 1 \). Solve for the even and odd mode impedance to give
$$Z_{0e} = Z_0 \left[ 1 + jZ_0 + (jZ_0)^2 \right]$$

$$Z_{0o} = Z_0 \left[ 1 - jZ_0 + (jZ_0)^2 \right]$$
The shunt impedance

\[ Z_{12} = \frac{jZ_0}{\sin 2\theta} = \frac{jZ_0}{\sin \pi (1 + \Delta \omega/\omega_0)} = \frac{-jZ_0\omega_0}{\pi (\omega - \omega_0)} \]

For a parallel LC circuit near resonance

\[ Z = \frac{-jL\omega_0^2}{2(\omega - \omega_0)} \quad \therefore \quad L = \frac{2Z_0}{\pi \omega_0}, \quad C = \frac{1}{\omega_0 L} = \frac{\pi}{2Z_0\omega_0} \]

Matching to the bandpass prototype, we have

\[ J_1 Z_0 = \left( \frac{C_1 L_1'}{L_1 C_1'} \right)^{\frac{1}{4}} = \sqrt[4]{\frac{\pi \Delta}{2 g_1}} \]

\[ J_2 Z_0 = J_1 Z_0^2 \left( \frac{C_2 L_2'}{L_2 C_2'} \right)^{\frac{1}{4}} = \sqrt[4]{\frac{\pi \Delta}{2 g_1 g_2}} \]

\[ J_3 Z_0 = \frac{J_2}{J_1} \sqrt[4]{\frac{\pi \Delta}{2 g_2}} \]

In general:

\[ J_1 Z_0 = \sqrt[4]{\frac{\pi \Delta}{2 g_1}} \]

\[ J_n Z_0 = \frac{\pi \Delta}{2 \sqrt{g_{n-1} g_n}}, \quad n=2,3,\ldots,N, \]

\[ J_{N+1} Z_0 = \sqrt[4]{\frac{\pi \Delta}{2 g_N}} \]
TABLE 8.8  Ten Canonical Coupled Line Circuits

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Image Impedance</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Circuit Image" /></td>
<td>(Z_0 = \frac{2Z_L Z_{in} \cos \theta}{\sqrt{(Z_L + Z_{in})^2 \cos^2 \theta - (Z_L - Z_{in})^2}})</td>
<td><img src="image2" alt="Response Graph" /></td>
</tr>
<tr>
<td><img src="image3" alt="Circuit Image" /></td>
<td>(Z_0 = \frac{2Z_L Z_{in} \sin \theta}{\sqrt{(Z_L - Z_{in})^2 - (Z_L + Z_{in})^2 \cos^2 \theta}})</td>
<td><img src="image4" alt="Response Graph" /></td>
</tr>
<tr>
<td><img src="image5" alt="Circuit Image" /></td>
<td>(Z_0 = \frac{(Z_L - Z_{in})^2 - (Z_L + Z_{in})^2 \cos^2 \theta}{2 \sin \theta})</td>
<td><img src="image6" alt="Response Graph" /></td>
</tr>
<tr>
<td><img src="image7" alt="Circuit Image" /></td>
<td>(Z_0 = \frac{Z_L - Z_{in}}{Z_L + Z_{in}})</td>
<td>All pass</td>
</tr>
<tr>
<td><img src="image8" alt="Circuit Image" /></td>
<td>(Z_0 = \frac{2Z_L Z_{in}}{Z_L + Z_{in}})</td>
<td>All pass</td>
</tr>
<tr>
<td><img src="image9" alt="Circuit Image" /></td>
<td>(Z_0 = \sqrt{Z_L Z_{in}})</td>
<td>All pass</td>
</tr>
<tr>
<td><img src="image10" alt="Circuit Image" /></td>
<td>(Z_0 = \frac{2Z_L Z_{in}}{Z_L - Z_{in}} \cot \theta)</td>
<td>All stop</td>
</tr>
<tr>
<td><img src="image11" alt="Circuit Image" /></td>
<td>(Z_0 = \sqrt{Z_L Z_{in}} \tan \theta)</td>
<td>All stop</td>
</tr>
<tr>
<td><img src="image12" alt="Circuit Image" /></td>
<td>(Z_0 = -\sqrt{Z_L Z_{in}} \cot \theta)</td>
<td>All stop</td>
</tr>
</tbody>
</table>

Table 8.8

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Example 8.7 COUPLED LINE BANDPASS FILTER DESIGN

$f_0=2\,\text{GHz}$, $\frac{\Delta f}{f_0}=10\%$, $Z_0=50\,\Omega$, $N=3$, 0.5 dB equal ripple.

<table>
<thead>
<tr>
<th>n</th>
<th>$g_n$</th>
<th>$Z_0J_n$</th>
<th>$Z_{0e}\Omega$</th>
<th>$Z_{0o}\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5963</td>
<td>0.3137</td>
<td>70.61</td>
<td>39.24</td>
</tr>
<tr>
<td>2</td>
<td>1.0967</td>
<td>0.1187</td>
<td>56.64</td>
<td>44.77</td>
</tr>
<tr>
<td>3</td>
<td>1.5963</td>
<td>0.1187</td>
<td>56.64</td>
<td>44.77</td>
</tr>
<tr>
<td>4</td>
<td>1.0000</td>
<td>0.3137</td>
<td>70.61</td>
<td>39.24</td>
</tr>
</tbody>
</table>
Bandpass Filters Using Capacitive Coupled Resonators

\[ \phi_i = -\tan^{-1}(2Z_0B_i) \]

\[ B_i = \frac{J_i}{1 - (Z_0J_i)^2} \]

\[ \theta_i = \pi - \frac{1}{2} \left[ \tan^{-1}(2Z_0B_i) + \tan^{-1}(2Z_0B_{i+1}) \right] \]
Example 8.10 CAPACITIVELY COUPLED SERIES RESONATOR BANDPASS FILTER DESIGN

\[ f_0 = 2 \text{GHz}, \quad \frac{\Delta f}{f_0} = 10\%, \quad Z_0 = 50\Omega, \quad N = 3, \quad 0.5 \text{ dB equal ripple.} \]

<table>
<thead>
<tr>
<th>n</th>
<th>( g_n )</th>
<th>( Z_0 J_n )</th>
<th>( B_n )</th>
<th>( C_n (\mu F) )</th>
<th>( \theta_n (^{\circ}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5963</td>
<td>0.3137</td>
<td>( 6.96 \times 10^{-3} )</td>
<td>0.554</td>
<td>155.8</td>
</tr>
<tr>
<td>2</td>
<td>1.0967</td>
<td>0.1187</td>
<td>( 2.41 \times 10^{-3} )</td>
<td>0.192</td>
<td>166.5</td>
</tr>
<tr>
<td>3</td>
<td>1.5963</td>
<td>0.1187</td>
<td>( 2.41 \times 10^{-3} )</td>
<td>0.192</td>
<td>155.8</td>
</tr>
<tr>
<td>4</td>
<td>1.0000</td>
<td>0.3137</td>
<td>( 6.96 \times 10^{-3} )</td>
<td>0.554</td>
<td></td>
</tr>
</tbody>
</table>
Bandpass Filters Using Capacitively Coupled Shunt Resonators

Figure 8.52
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Figure 8.53
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Similar to 8.8 bandstop filter,

\[ Z_{0n} = \frac{\pi Z_0 \Delta}{4g_n} \]

\[ J_{01} Z_0 = \sqrt{\frac{\pi \Delta}{4g_1}} \]

\[ J_{n,n+1} Z_0 = \sqrt{\frac{\pi \Delta}{4g_n g_{n+1}}} \]

\[ J_{N,N+1} Z_0 = \sqrt{\frac{\pi \Delta}{4g_N g_{N+1}}} \]

From Fig. 8.38

\[ C_{01} = \frac{J_{01}}{\omega_0 \sqrt{1 - (Z_0 J_0)^2}} \]

\[ C_{n,n+1} = \frac{J_{n,n+1}}{\omega_0} \]

\[ C_{N,N+1} = \frac{J_{N,N+1}}{\omega_0 \sqrt{1 - (Z_0 J_{N,N+1})^2}} \]

\[ C_n' = C_n + \Delta C_n = C_n - C_{n-1,n} - C_{n,n+1} \]

\[ \ell_n = \frac{\lambda}{4} + \left( \frac{Z_0 \omega_0 \Delta C_n}{2\pi} \right) \lambda \]
Example 8.10 CAPACITIVELY COUPLED SHUNT RESONATOR BANDPASS FILTER DESIGN

N=3, 0.5 dB equal-ripple, $f_0=2.5\text{GHz}$, $\frac{\Delta f}{f_0}=10\%$, $Z_0=50\Omega$.

<table>
<thead>
<tr>
<th>n</th>
<th>$g_n$</th>
<th>$Z_0J_{n-1,n}$</th>
<th>$C_{n-1,n},(\text{pF})$</th>
<th>$\Delta C_n,(\text{pF})$</th>
<th>$\Delta \ell_n,(\lambda)$</th>
<th>$\ell,(^\circ)$</th>
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<td>0.2218</td>
<td>0.2896</td>
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<td>-0.04565</td>
<td>73.6</td>
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<td>0.0594</td>
<td>0.0756</td>
<td>-0.1512</td>
<td>-0.0189</td>
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<td>0.2896</td>
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