A sliding-mode controller (SMC) is proposed for semi-active suspensions to achieve ride comfort and handling performance simultaneously. First, a nonlinear quarter-car model of Macpherson strut suspension is established in Matlab/Simulink. Constrained damper force and actuator dynamics are considered for the damper model. System identification is applied to the nonlinear model for obtaining the linear model parameters. Kalman filter is designed based on the linear model and the actuator dynamics to estimate the state responses required for SMC. The sliding surface consists of tire deflection and sprung mass acceleration. The proposed SMC is evaluated using the nonlinear model for both time and frequency domain responses. Robustness due to the increased sprung mass and deteriorated suspension is also investigated in this paper. Preliminary simulation results show improved ride comfort without sacrificing the road holding performance.

Keywords: semi-active suspension; sliding-mode control; actuator dynamics; Kalman filter

1. Introduction

In order to achieve ride comfort and handling performance simultaneously, compromises are often made for the conventional passive suspension. These compromises can be changed for semi-active and active suspensions by including adjustable damper and active actuators, respectively.

Williams [1,2] published a fairly complete review of automotive active suspensions. Active suspensions can provide not only ride comfort for straight line driving, but also attitude holding for both steady-state and transient pitch and roll motions. Although semi-active suspension might not provide ride comfort as good as active suspension, and can only handle transient pitch and roll motions, it only

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consumes small amounts of power and does not require heavy and expensive power supply.

Various control methodologies have been proposed for semi-active suspensions in the literature. Karnopp et al. [3] proposed a simple on-off control strategy to switch between the maximum and minimum damping settings according to the multiplication of sprung mass acceleration and suspension stroke velocity. Their approach can effectively suppress the displacement, velocity, and acceleration of the sprung mass. Al-Holou et al. [4] proposed a skyhook control logic by dividing the frequency of the wheel displacement and damping settings into six regions and four modes, respectively. Measured sprung mass acceleration and estimated unsprung mass displacement are used to modulate the damping settings. Their simulation results showed improvement of ride comfort and handling performance over the passive suspensions. Savarese et al. [5] compared the performance of three strategies: two-state skyhook damping, linear skyhook damping, and their approach based on two-state switching dampers. Their frequency responses showed less sprung mass acceleration over the other two strategies. Hong et al. [6] proposed a control using pole placement method. Because the model used for their controller design is based on the linearization of a nonlinear Macpherson strut suspension, their approach can obtain the damping force which is more suitable for the real configuration. Motta et al. [7] compared the performance of three systems: optimized passive system, semi-active on-off system, and semi-active continuously variable damper system. Their simulation results showed that the semi-active continuously variable damper system achieves better ride comfort, with higher tire deflection when compared with the passive and semi-active on-off systems. Mo et al. [8] proposed a bistate control algorithm based on a consistent model of the hydrodynamics of the actuator. Their
experimental results showed a perceivable reduction of sprung mass acceleration and tire force variation relative to the passive design.

Yokoyama et al. [9] proposed a sliding-mode control (SMC) based on the theory of model following control. A desired semi-active suspension system is chosen as the reference model to be followed. Their approach only achieved the desired performance of sprung mass acceleration in the low frequency range, not the high frequency range due to chattering. Lam and Liao [10] proposed a SMC with the sliding surface which consists of the tracking errors of the sprung mass displacement and velocity. Both simulation and experimental results showed improvement of the ride comfort. Han et al. [11] proposed a robust SMC with the sliding surface which consists of sprung mass displacement and velocity, and unsprung mass displacement and velocity for suppressing the vibration level of each state. Their simulation results showed reduced transmissibility of suspension travel and sprung mass acceleration around the body frequency. Yao and Zheng [12] proposed a model reference SMC which utilizes an approximate ideal skyhook system as a reference model. Their sliding surface is similar to the one in [10]. Their simulation results showed better ride quality and handling performance over other controllers. Zheng et al. [13] designed a fuzzy SMC with the sliding surface which consists of sprung mass acceleration, suspension stroke, and tire deflection. Their simulation results showed ride comfort improvement and good robustness with respect to the parameter uncertainty. Cheng et al. [14] proposed a SMC with the sliding surface which consists of the tracking errors of the sprung mass displacement and velocity, and the integral of the tracking error of the sprung mass displacement. They introduced neural network to optimize the SMC result. Their simulation results showed effective improvement of sprung mass acceleration.
A SMC is proposed for semi-active suspensions in this paper. In order to achieve ride comfort and handling performance simultaneously, the sliding surface is designed to include sprung mass acceleration and tire deflection. Constrained damper force and actuator dynamics are considered for the damper model. System identification is applied to the nonlinear quarter-car model of Macpherson strut suspension for obtaining the linear model parameters. Kalman filter is designed based on the linear model and the actuator dynamics to estimate the state responses required for SMC. The nonlinear model is used to evaluate the proposed SMC for both time and frequency domain responses. Robustness due to the increased sprung mass and deteriorated suspension is also investigated in this paper.

The remainder of this paper is organized as follows. The nonlinear and linear quarter-car models are introduced in Section 2, followed by the proposed algorithm in Section 3. Simulation results are presented in Section 4. Finally, conclusions are made in Section 5.

2. Quarter-car Model

2.1 Nonlinear Model

The Macpherson strut suspension as shown in Figure 1 is used as the nonlinear model in this paper. Hong et al. [6] derived the equations of motion as follows.
\begin{align}
(m_s + m_u)\ddot{z}_s + m_u l_c \cos(\theta - \theta_0)\dot{\theta} - m_u l_c \sin(\theta - \theta_0)\dot{\theta}^2
\end{align}
\begin{align}
+k_u \left\{ z_s + l_c [\sin(\theta - \theta_0) - \sin(-\theta_0)] - z_r \right\} = 0
\end{align}
\begin{align}
m_u l_c^2 \ddot{\theta} + m_u l_c \cos(\theta - \theta_0)\dot{z}_s + \frac{s_c m c b^1 \sin^2(\alpha' - \theta)}{4(a_i - b_i \cos(\alpha' - \theta))} \dot{\theta}
\end{align}
\begin{align}
+k_u l_c \cos(\theta - \theta_0) \left\{ z_s + l_c [\sin(\theta - \theta_0) - \sin(-\theta_0)] - z_r \right\}
\end{align}
\begin{align}
+ \frac{1}{2} k_u \sin(\alpha' - \theta) \left[ \frac{d_i}{\sqrt{c_i - d_i \cos(\alpha' - \theta)}} - b_i \right] = 0
\end{align}

where $m_s$ is the sprung mass; $m_u$ is the unsprung mass; $k_s$ and $k_u$ are the spring constants for the suspension and tire, respectively; $c_s$ is the damping coefficient of the suspension; $z_s$ is the displacement of the sprung mass; $z_u$ is the displacement of the unsprung mass; $z_o$ is the road profile input; and the subscript $n$ denotes for parameters associated the nonlinear model. $s_c$ is the scaling factor for adjusting the damping coefficient. $\theta$ is the angular displacement of the control arm. $\theta_0$ is the initial angular displacement of the control arm. $l_A$ and $l_B$ denote for distances between point $O$ to point $A$ and $B$, respectively. $\alpha' = \alpha + \theta_0$; $a_i = l_A^2 + l_B^2$; $b_i = 2l_A l_B$; $c_i = a_i^2 - a_i b_i \cos \alpha'$; and $d_i = a_i b_i - b_i^2 \cos \alpha'$.

A first order system is used to simulate the adjustable damper dynamics as follows.
\begin{align}
\tau \dot{s}_c + s_c = s_m
\end{align}

where $s_m$ is the desired scaling factor from the controller; $\tau$ is the time constant and is set to be 15 ms [15] in this paper. The overall damping force of the suspension $f_d$ is limited by the available damping forces as follows.
\begin{align}
f_d = \begin{cases}
f_{d,\text{max}} & \text{sgn}(\dot{z}_s - \dot{z}_u), \\
 s_c m \left| \dot{z}_s - \dot{z}_u \right| & \geq f_{d,\text{max}} \\
 s_c m \left| \dot{z}_s - \dot{z}_u \right| & < f_{d,\text{max}} \end{cases}
\end{align}
\begin{align}
f_{d,\text{min}} & \leq \begin{cases}
f_{d,\text{max}} & \text{sgn}(\dot{z}_s - \dot{z}_u), \\
 s_c m \left| \dot{z}_s - \dot{z}_u \right| & \leq f_{d,\text{min}} \\
 s_c m \left| \dot{z}_s - \dot{z}_u \right| & < f_{d,\text{max}} \end{cases}
\end{align}
where \( \text{sgn} \) is the sign function; \( f_{d, \text{max}} \) and \( f_{d, \text{min}} \) are the maximum and minimum available damping forces, respectively.

### 2.2 Linear Model

The linear model as shown in Figure 2 is used for controller design in this paper. The equations of motion can be expressed as follows.

\[
\begin{align*}
\dot{z}_s + k_s (z_s - z_{us}) + c_s (\dot{z}_s - \dot{z}_{us}) &= -f_c \\
m_{us} \ddot{z}_{us} + k_{us} (z_{us} - z_0) - k_s (z_s - z_{us}) - c_s (\dot{z}_s - \dot{z}_{us}) &= f_c
\end{align*}
\]

where \( f_c \) is the control force input.

Similar to Equation (3), a first order system is used to simulate the control force dynamics as follows.

\[
\tau \dot{f}_c + f_c = f_{in}
\]

where \( f_{in} \) is the control force command from the controller.

If we define the state vector \( x = [z_{us} - z_0 \quad \dot{z}_{us} \quad z_s - z_{us} \quad \dot{z}_s \quad f_c]^T \), the control input \( u = f_{in} \), and the disturbance \( w = \dot{z}_0 \), we can obtain the state-space model from Equations (5), (6), and (7) as follows.

\[
\dot{x} = Ax + Bu + Gw
\]
In order to obtain the parameters for the linear quarter-car model, Kim et al. [16] assumed different damping coefficients for extension and compression, and applied the least square method for system identification. We assumed the same damping coefficients for both directions to simplify the derivation. By setting \( f_c = 0 \), i.e. passive suspension, we can rewrite Equations (5) and (6) as follows.

\[
\begin{align*}
\dot{m}_s \ddot{z}_s + k_s (z_s - z_{us}) + c_s (\dot{z}_s - \dot{z}_{us}) &= \phi^T \theta \\
\dot{m}_{us} \ddot{z}_{us} - k_s (z_s - z_{us}) - c_s (\dot{z}_s - \dot{z}_{us}) - f_t &= \phi^T \theta - f_t,
\end{align*}
\]

where \( f_t = k_s (z_{us} - z_o) \) is the tire force; \( \theta = [m_s \ m_{us} \ k_s \ c_s]^T \) is the parameter vector; \( \phi_1^T = [\ddot{z}_s \ 0 \ z_s - z_{us} \ \dot{z}_s - \dot{z}_{us}] \) and \( \phi_2^T = [0 \ \ddot{z}_{us} \ -(z_s - z_{us}) \ -(\dot{z}_s - \dot{z}_{us})] \) are regression vectors.

The error cost function \( E \) can be defined as follows.

\[
E = \sum_{i=1}^{N} [\epsilon_i^2(i) + \epsilon_2^2(i)]
\]

where \( \epsilon_1 = \phi_1^T \hat{\theta} \) and \( \epsilon_2 = \phi_2^T \hat{\theta} - f_t \) are estimated errors; \( \hat{\theta} = [\hat{m}_s \ \hat{m}_{us} \ \hat{k}_s \ \hat{c}_s]^T \) is the estimated parameter vector; \( i \) denotes for the \( i^{th} \) measurement; and \( N \) is the total number of measurements. \( E \) can be minimized if the following condition is satisfied.

\[
\frac{\partial E}{\partial \theta} = \sum_{i=1}^{N} [\phi_1(i)\phi_1^T(i) + \phi_2(i)\phi_2^T(i)]\hat{\theta} = \sum_{i=1}^{N} \phi(i)f_t(i) = 0
\]
We can then obtain \( \hat{\theta} \) for the linear model as follows.

\[
\hat{\theta} = \left( \sum_{i=1}^{N} [\phi(i)\phi^T(i) + \phi_i(i)\phi_i^T(i)] \right)^{-1} \sum_{i=1}^{N} \phi_i(i)f_i(i)
\]  

(13)

3. Controller Design

3.1 On-off Control

On-off control strategy [3] is used as the baseline controller for semi-active suspension. The damping coefficient switches between the maximum and minimum damping settings according to the multiplication of sprung mass acceleration and suspension stroke velocity as shown below.

\[
c_s = \begin{cases} 
    c_{\text{max}}, & \ddot{z}_s (\dot{z}_s - \dot{z}_{\text{ass}}) > 0 \\
    c_{\text{min}}, & \ddot{z}_s (\dot{z}_s - \dot{z}_{\text{ass}}) \leq 0 
\end{cases}
\]

(14)

3.2 Sliding Mode Control

In order to achieve ride comfort and road holding at the same time, the sliding surface \( s \) consists of the sprung mass acceleration and tire deflection as follows.

\[
s = \ddot{z}_s + \rho (z_{\text{ass}} - z_0) = \ddot{x}_4 + \rho x_1
\]

(15)

where \( \rho \) is a positive weighting factor. In order to satisfy the sliding condition, we need

\[
s\dot{s} \leq -\eta |s|
\]

(16)

where \( \eta \) is a positive real number. The derivative of the sliding surface \( s \) along the trajectories of Equation (8) is given by

\[
\dot{s} = \ddot{x}_4 + \rho \dot{x}_1 = a_{42}\ddot{x}_2 + a_{43}\ddot{x}_3 + a_{44}\ddot{x}_4 + a_{45}x_3 + a_{46}\dot{b}_3 u + \rho \dot{x}_1
\]

(17)
where $a_{ij}$ denotes for the element at the $i^{th}$ row and $j^{th}$ column of the system matrix $A$. $b_j$ denotes for the $j^{th}$ element of the input matrix $B$. $x_i$ denotes for the $i^{th}$ element of the state vector.

The control command can then be expressed as follows.

$$ u = -\frac{1}{a_{45}b_5} (a_{42}\dot{x}_2 + a_{43}\dot{x}_3 + a_{44}\dot{x}_4 + a_{45}\dot{x}_5 + \rho \dot{x}_1 + K \text{sgn}(x)) $$ (18)

where $K$ denotes for the control discontinuity. The hat denotes for the perceived parameters which maybe different from the true parameters. By substituting Equation (18) into Equation (17), we can then obtain $K$ as follows.

$$ K \geq \frac{1}{\beta} \left[ (a_{42} - \beta \dot{a}_{42}) \dot{x}_2 + (a_{43} - \beta \dot{a}_{43}) \dot{x}_3 + (a_{44} - \beta \dot{a}_{44}) \dot{x}_4 + (a_{45}\dot{a}_{55} - \beta \dot{a}_{45}\dot{a}_{55}) x_5 \right] + |\rho||(1-\beta)\dot{x}_1| + \eta $$ (19)

where $\beta = \frac{a_{45}b_5}{a_{45}\dot{a}_{45}}$. After obtaining the control command $u$, the total damping force can be expressed as follows.

$$ f_d = c_r (\dot{z}_r - \dot{z}_w) + u $$ (20)

We can then obtain the desired damping coefficient as follows.

$$ c_d = \begin{cases} 
1, & \text{for } \dot{z}_s - \dot{z}_w = 0 \\
\max \left( 0, \frac{c_r (\dot{z}_s - \dot{z}_w) + u}{\dot{z}_s - \dot{z}_w} \right), & \text{for } \dot{z}_s - \dot{z}_w \neq 0
\end{cases} $$ (21)

The desired scaling factor for adjusting the damping coefficient of the nonlinear model can then expressed as follows.

$$ s_{in} = \frac{c_d}{c_r} $$ (22)

### 3.3 Kalman Filter
Since some of the states required for computing the control command are not measurable, a Kalman filter is designed to provide state estimates in this paper. First, we can apply zero-order-hold transformation to obtain the discrete state-space model of Equation (8) as follows.

\[
x(k+1) = \Phi x(k) + \Gamma_u u(k) + \Gamma_w w(k)
\]  

where \( k \) denotes for the \( k^{th} \) sample data; \( \Phi \), \( \Gamma_u \), and \( \Gamma_w \) are the system matrix, control input matrix, and disturbance input matrix in the discrete domain, respectively.

If the sprung mass acceleration and suspension stroke are measurable, we can obtain the output equation as follows.

\[
y(k) = Hx(k) + v(k)
\]

where \( v \) is the measurement noise vector; \( H \) is the output matrix and can be expressed as

\[
H = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & a_{41} & a_{42} \\
0 & 0 & 0 & a_{43} & a_{44} \\
0 & 0 & 0 & a_{45} & a_{46}
\end{bmatrix}
\]

Because the observability matrix has full rank, the discrete system is fully observable. We can then obtain the Kalman filter as follows.

\[
\hat{x}(k+1) = \Phi \hat{x}(k) + \Gamma_u u(k) + L(y(k) - \hat{y}(k))
\]

\[
\hat{y}(k) = H\hat{x}(k)
\]

where \( \hat{x} \) and \( \hat{y} \) are the estimated state and output vectors, respectively. \( L \) is the estimation gain matrix, which can be designed as

\[
L = P(k)H^T R^{-1}
\]

where \( P(k) \) is the estimate error covariance matrix as

\[
P(k) = M(k) - M(k)H^T (HM(k)H^T + R)^{-1}HM(k)
\]
and $R = E[v(k)v^T(k)]$ is the covariance matrix of the measurement noise $v$. $M(k)$ is the update law for $P(k)$ as

$$M(k+1) = \Phi P(k)\Phi^T + \Gamma_w Q \Gamma_w^T$$  \hspace{1cm} (30)$$

where $Q = E[w(k)w^T(k)]$ is the process noise variance. After obtaining the estimated state vector, variables of $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$, $\hat{x}_4$, and $\hat{x}_5$ can then be expressed in terms of $\hat{x}$ using Equation (8). We can then compute the control command $u$ as follows.

$$u = -\frac{1}{\hat{a}_{42}\hat{b}_5} \left( \hat{a}_{42}\hat{x}_2 + \hat{a}_{43}\hat{x}_3 + \hat{a}_{44}\hat{x}_4 + \hat{a}_{45}\hat{x}_5 + \rho \hat{x}_1 + K \text{ sgn}(s) \right)$$  \hspace{1cm} (31)$$

If the errors of the estimated states are bounded, we can express $\Delta x_1$, $\Delta x_2$, $\Delta x_3$, $\Delta x_4$, and $\Delta x_5$ as follows.

$$\hat{x}_i = \hat{x}_i + \Delta \hat{x}_i, \quad \hat{x}_2 = \hat{x}_2 + \Delta \hat{x}_2, \quad \hat{x}_3 = \hat{x}_3 + \Delta \hat{x}_3, \quad \hat{x}_4 = \hat{x}_4 + \Delta \hat{x}_4, \quad \text{and} \quad \hat{x}_5 = x_5 + \Delta x_5$$  \hspace{1cm} (32)$$

where $\Delta \hat{x}_1$, $\Delta \hat{x}_2$, $\Delta \hat{x}_3$, $\Delta \hat{x}_4$, and $\Delta x_5$ are bounded as follows.

$$|\Delta \hat{x}_1| \leq \gamma_1, \quad |\Delta \hat{x}_2| \leq \gamma_2, \quad |\Delta \hat{x}_3| \leq \gamma_3, \quad |\Delta \hat{x}_4| \leq \gamma_4, \quad \text{and} \quad |\Delta x_5| \leq \gamma_5$$  \hspace{1cm} (33)$$

By substituting Equations (31) and (32) into Equation (17), we can obtain $K$ as shown below to satisfy the sliding condition if there exists the errors of the estimated states.

$$K \geq \frac{1}{\beta} \left[ (a_{42} - \beta \hat{a}_{42}) \hat{x}_2 + (a_{43} - \beta \hat{a}_{43}) \hat{x}_3 + (a_{44} - \beta \hat{a}_{44}) \hat{x}_4 + (a_{45} - \beta \hat{a}_{45}) \hat{x}_5 + \rho \left[ (1 - \beta) \hat{x}_1 + \eta \right] \right. \left. + \gamma_2 \left. + \gamma_3 \right. + \gamma_4 \left. + \hat{a}_{45} \gamma_5 + \rho \gamma_1 \right]$$  \hspace{1cm} (34)$$

4. Simulation Results

The nonlinear model is used to evaluate the proposed estimator and SMC for both time and frequency domain responses. Parameter values used for simulation are listed
in Table 1. Robustness due to the increased sprung mass and deteriorated suspension is also investigated in this paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_s$</td>
<td>damping coeff. of the linear model</td>
<td>N-s/m</td>
<td>1410.65</td>
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<tr>
<td>$c_{\text{max}}$</td>
<td>maximum damping coeff. of the linear model</td>
<td>N-s/m</td>
<td>1953.21</td>
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<tr>
<td>$c_{\text{min}}$</td>
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<td>N-s/m</td>
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<tr>
<td>$c_{\text{on}}$</td>
<td>damping coeff. of the nonlinear passive suspension</td>
<td>N-s/m</td>
<td>1950</td>
</tr>
<tr>
<td>$c_{\text{on max}}$</td>
<td>maximum damping coeff. of the nonlinear model</td>
<td>N-s/m</td>
<td>2700</td>
</tr>
<tr>
<td>$c_{\text{on min}}$</td>
<td>minimum damping coeff. of the nonlinear model</td>
<td>N-s/m</td>
<td>1400</td>
</tr>
<tr>
<td>$k_s$</td>
<td>suspension spring constant of the linear model</td>
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<td>12948.12</td>
</tr>
<tr>
<td>$k_{on}$</td>
<td>suspension spring constant of the nonlinear model</td>
<td>N/m</td>
<td>17658</td>
</tr>
<tr>
<td>$k_{us}$</td>
<td>tire spring constant of the linear model</td>
<td>N/m</td>
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</tr>
<tr>
<td>$k_{usn}$</td>
<td>tire spring constant of the nonlinear model</td>
<td>N/m</td>
<td>183887</td>
</tr>
<tr>
<td>$m_s$</td>
<td>sprung mass of the linear model</td>
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<tr>
<td>$m_{on}$</td>
<td>sprung mass of the nonlinear model</td>
<td>Kg</td>
<td>453</td>
</tr>
<tr>
<td>$m_{us}$</td>
<td>unsprung mass of the linear model</td>
<td>Kg</td>
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</tr>
<tr>
<td>$m_{usn}$</td>
<td>unsprung mass of the nonlinear model</td>
<td>Kg</td>
<td>36</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time constant of the actuator</td>
<td>ms</td>
<td>15</td>
</tr>
</tbody>
</table>

4.1 Estimation Results

A sinusoidal-shape speed bump [17] with the height of 101.6 mm and the length of 3.65 m is used to evaluate the performance of the proposed estimator under the vehicle speed of 8.9 m/s. The simulation results of the estimated tire deflection, unsprung mass velocity, suspension stroke, and sprung mass velocity are shown in Figure 3. The proposed estimator can provide accurate estimations of unsprung mass velocity, suspension stroke, and sprung mass velocity. Since the road profile is the unknown disturbance in this paper, it results in larger estimation error for the tire deflection.
4.2 Time-domain Responses

The sinusoidal-shape speed bump is also used to evaluate the performance of the proposed control under the vehicle speed of 8.9 m/s. Responses of tire deflection, suspension stroke, and sprung mass acceleration are shown in Figures 4, 5, and 6, respectively.
Figure 4. Responses of tire deflection for going over a speed bump

Figure 5. Responses of suspension stroke for going over a speed bump
The tire deflection changes from compression to extension when going over the speed bump. Both on-off control and proposed SMC can suppress 20% tire deflection during compression. During extension, only the proposed SMC can suppress the 38.3% tire deflection; on-off control actually has tire deflection larger than the passive suspension about 26.8%. After going over the speed bump, the on-off control can regulate the tire deflection back to the equilibrium point faster than the passive suspension and the proposed SMC. Similar to the responses of the tire deflection, both on-off control and proposed SMC can suppress 15% sprung mass acceleration while going over the speed bump. During extension, the proposed SMC can suppress the 27.3% sprung mass acceleration; on-off control has sprung mass acceleration larger than the passive suspension about 20.4%. However, the proposed SMC shows almost no effect for the suspension stroke, while the on-off control can suppress the suspension stroke during extension.

Responses of the scaling factor for adjusting the damping coefficient of the nonlinear model are shown in Figure 7. The proposed SMC can adjust the damping more appropriately than the on-off control. The on-off control still maintains
maximum damping coefficient around 2.85 sec and results in performance worse than the passive suspension for road holding and ride comfort.

4.3 Frequency-domain Responses

ISO 2631 criterion presents three limits for vertical vibrations. Human body is most sensitive to vibrations between 4 to 8 Hz. Frequency responses are conducted using fast Fourier transform with a sinusoidal sweep input of road profile to evaluate the performance of the proposed SMC. Responses of tire deflection, suspension stroke, and sprung mass acceleration are shown in Figures 8, 9, and 10, respectively.
Figure 8. Frequency responses of tire deflection

Figure 9. Frequency responses of suspension stroke
Figure 10. Frequency responses of sprung mass acceleration

Both on-off and proposed controllers can reduce tire deflection, suspension stroke, and sprung mass acceleration simultaneously around the body frequency. The on-off control can not suppress the sprung mass acceleration between 5 to 9 Hz. On the contrary, the proposed SMC can reduce the sprung mass acceleration from the body frequency up to 9 Hz while keeping the tire deflection smaller than that of the passive suspension for frequencies below 6 Hz. However, the proposed SMC can not suppress the suspension stroke for frequencies higher than 3 Hz, while the on-off control can reduce the suspension stroke around the wheel hop frequency.

4.4 Robustness Study

In order to verify the robustness of the proposed SMC for frequency responses, the sprung mass is increased by 30%, the suspension spring constant is reduced by 20%, and the time constant of the actuator is increased by 20%. Frequency responses of tire deflection, suspension stroke, and sprung mass acceleration are shown in Figures 11, 12, and 13, respectively. Similar to the results in Section 4.2, the proposed SMC can
still reduce the sprung mass acceleration without increasing the tire deflection under the presence of parameter uncertainty.

Figure 11. Frequency responses of tire deflection for robustness study

Figure 12. Frequency responses of suspension stroke for robustness study
5. Conclusions

A SMC is proposed for semi-active suspension with actuator dynamics in this paper. Constrained damper force and actuator dynamics are considered for the damper model. The proposed SMC is designed based on a linear quarter-car model with actuator dynamics. Parameters of the linear model are obtained from the system identification of a nonlinear Macpherson strut suspension model. Kalman filter is designed based on the linear model with actuator dynamics to provide state estimates required for SMC. The sliding surface consists of tire deflection and sprung mass acceleration for road holding and ride comfort, respectively. The nonlinear model is used to evaluate the performance of the proposed SMC. The proposed SMC can suppress the tire deflection and sprung mass acceleration while going over the speed bump. For frequency responses, the proposed SMC can reduce the sprung mass acceleration without increasing the tire deflection. Under the presence of parameter uncertainties due to the increased sprung mass and deteriorated suspension, desired performance can still be achieved for the proposed SMC.
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References
