The Distinctiveness of a Curve in a Parameterized Neighborhood: Extraction and Applications

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Design and computational Framework
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Motivation

- *The weak signal problem*: Want to pick up weak signals in the presence of strong signals.

- Example: applying the Hough transform to an image that contains a weak signal and a strong signal simultaneously.

- A false positive induced by the strong signal can receive more votes than the weak signal.
  - Due to peak spreading/splitting, multi-signal/noise interference, etc.
The task: Detect the two concentric circles in the image
Hough transform picks up many populous hypotheses around the strong signal

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>(134, 134, 59)</td>
<td>signal (inner circle)</td>
</tr>
<tr>
<td>(134, 135, 59)</td>
<td>ghost</td>
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<td>(135, 134, 73)</td>
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<td>(134, 133, 74)</td>
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</tr>
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Ranked by number of pixels
The weak signal problem aggravates as noise increases.

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10% noise

30% noise
A feature with more distinguishing power

- The pixel count feature does not work.
- Any feature that allows us to solve the weak signal problem?
  - One that gives higher feature values to a signal over its false positives.
  - One that gives higher feature value to a weak signal over false positives induced by other signals.
The idea: distinctiveness

- Manifestation of a hypothesis should be
  - Crisp (high feature values) if a signal;
  - Fuzzy (not-so-high feature values) if a *ghost* of a signal (i.e., signal look-alike);
  and
  - Non-distinctive (low feature value) if it is relatively unrelated to a signal
- Manifestation is a *local phenomenon* around the neighborhood of the curve
Distinctiveness feature illustrated

- **Exact**: signal falls into one bin
- **Ghost**: signal spreads into neighboring bins
- **Random**: signal cuts up flatly

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Elements for computing the new feature of a hypothesis

- Define a set of curves that are similar to the hypothesis in its neighborhood
  - *One parameter family around the hypothesis*
- Count the number of pixels received by each member of the set of curves
  - *Hough transform derived from the one-parameter family*
- Compute the distinctiveness index of the hypothesis
  - *Sample statistics on the Hough space*
The computational framework

Given a hypothesis \( h(x, y) = 0 \) found by a curve detector in an edge map \( I \):

S1 Define a one-dimensional Hough transform for extracting the distinctiveness feature:

S1.1 Select an analytic form for the one-parameter family for modeling the neighborhood of the hypothesis \( h(x, y) = 0 \).

S1.2 Determine the specific type of parameterization by specifying a minimal set of control points associated with \( h(x, y) = 0 \).

S1.3 Set the bin size and the number of bins for the Hough transform.

S2 Apply the Hough transform to edge pixels in edge map \( I \) to accumulate votes in the bins.

S3 From the bins in the accumulator, calculate the score of distinctiveness for \( h(x, y) = 0 \).
S1.1: One-parameter family of curves from the hypothesis

One parameter family

\[ \mathcal{F}(\lambda) \equiv (1 - \lambda)f_1(x, y) + \lambda f_2(x, y) = 0, \lambda \in \mathbb{R}, \]

Base curves are derived from hypothesis \( h(x, y) = 0 \):

\[ \mathcal{F}(0) \equiv f_1(x, y) = 0 \]
\[ \mathcal{F}(1) \equiv f_2(x, y) = 0 \]

Hypothesis is a member of the family:

\[ \mathcal{F}(\lambda_h) \equiv h = 0 \quad \text{for some} \quad \lambda_h \in \mathbb{R}. \]
Examples of one-parameter family of curves (1/2)

(a) a pencil of ellipses, (b) a system of ellipses with the same vertices
Examples of one-parameter family of curves (2/2)

(c) A concentric system of circles, and (d) a coaxal system of circles.
Constructing the base curves from the hypothesis

- Selecting a minimal set of control points associated with $h(x,y) = 0$ that constitutes $dof(h) - 1$ independent conditions
- Subjecting the one parameter family of curves to the constraints of the control points
S1.2: Hough transform derived from the one-parameter family

The mapping

\[ \lambda : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}, \]

\[ \lambda(x, y) = \frac{f_1(x, y)}{f_1(x, y) - f_2(x, y)}, \]

sends a point in the plane to a real value characterizing the curve that the point is on.
S1.3: Determine the parameters of the Hough transform

- Range of parameter
  - So that the hypothesis is at the center of the range

- Bin size
  - Just large enough so that all edges compatible with the hypothesis is in the central bin

- Number of bins
  - Just enough so that the parameter range captures a small neighborhood around the hypothesis
S2: apply the Hough transform

- Image points mapped into a one-dimensional histogram of points against parameter bins
S3: compute distinctiveness value

Let $X_h$ be the number of edge pixels compatible with the hypothesis $h(x, y) = 0$. The distinctiveness score for the hypothesis $h(x, y) = 0$ is calculated by

$$w(h) = \begin{cases} \frac{X_h - \bar{X}(h)}{S(h)} & \text{if } X_h > \bar{X}(h), \\ 0 & \text{otherwise}, \end{cases} \quad (5)$$

where the sample mean $\bar{X}(h)$ and the sample variance $S^2(h)$ except the hypothesis are calculated as follows [17]:

$$\bar{X}(h) = \sum_{i=0}^{m-1} \frac{X_i}{(m - 1)} \quad (6)$$

$$S^2(h) = \sum_{i=0}^{m-1} \frac{(X_i - \bar{X}(h))^2}{(m - 2)}. \quad (7)$$
Example: pencil of ellipses

(1/3)

Base curves:

\[ l_1 l_2 = 0 \text{ and } l_3 l_4 = 0. \]

Pencil of ellipses:

\[ \mathcal{F}(\lambda) \equiv (1 - \lambda)l_1 l_2 + \lambda l_3 l_4 = 0, \]

\[ \lambda(x, y) = \frac{l_1 l_2}{l_1 l_2 - l_3 l_4}. \]
Example: pencil of ellipses (2/3)

Given a hypothesis \( h = 0 \) of an ellipse described by

\[
\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} - 1 = 0 \text{ and } \theta,
\]

The control points:

\[
P_1 = (r \cos \theta - r \sin \theta + x_0, r \sin \theta + r \cos \theta + y_0),
\]
\[
P_2 = (-r \cos \theta - r \sin \theta + x_0, -r \sin \theta + r \cos \theta + y_0),
\]
\[
P_3 = (-r \cos \theta + r \sin \theta + x_0, -r \sin \theta - r \cos \theta + y_0),
\]
\[
P_4 = (r \cos \theta + r \sin \theta + x_0, r \sin \theta - r \cos \theta + y_0),
\]
Example: pencil of ellipses

(3/3)

The base curves:

\[
\begin{align*}
P_1P_2 & \equiv l_1 = (x - x_0) \sin \theta - (y - y_0) \cos \theta + r = 0, \\
P_3P_4 & \equiv l_2 = (x - x_0) \sin \theta - (y - y_0) \cos \theta - r = 0, \\
P_1P_4 & \equiv l_3 = (x - x_0) \cos \theta + (y - y_0) \sin \theta - r = 0, \\
P_2P_3 & \equiv l_4 = (x - x_0) \cos \theta + (y - y_0) \sin \theta + r = 0. 
\end{align*}
\]

The hypothesis:

\[
\mathcal{F} \left( \frac{b^2}{a^2 + b^2} \right) \equiv h = 0,
\]
A test image contains:
- Target: a weak signal
- Decoy: one or more strong signals
- Random noise

Features compared for their effectiveness:
- Pixel count (PCT)
- Distinctiveness based on Pencil of ellipse (PEHT)
Fig. 5. Two sample synthetic images of size $100 \times 100$: (a) a 70 percent-strength target and a full-strength decoy in the first experiment and (b) a 75 percent-strength target and a full-strength decoy at 25 percent uniform noise in the second experiment.
Performance with respect to signal strength
Performance with respect to noise level
Performance with respect to separation

Fig. 8. Test results of the second experiment: the distribution of tests against intervals of $\phi_T$ with signal strength at 90 percent and noise level at 20 percent.
Performance comparison on real image (1/3)
Performance comparison on real image (2/3)

<table>
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<th>score by PCT</th>
<th>Hypothesis</th>
<th>classification</th>
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<tbody>
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<td>signal (inner circle)</td>
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<td>ghost</td>
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<td>0.388</td>
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<td>0.357</td>
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### Performance comparison on real image (3/3)

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Concluding remarks

- Feature measuring the distinctiveness of curve in a parameterized neighborhood more effectively solves the weak signal problem than the pixel count feature.
- Computational framework for generating feature extractors for line, circle, and ellipse.
- Can be used as the tester in the “generate and test” problem solving paradigm for curve detection.
  - E.g., probabilistic Hough transform, RANSAC, k-RANSAC.